A DECOMPOSITION APPROACH FOR BATCH SCHEDULING IN THE PROCESS INDUSTRY

Christos T. Maravelias
Department of Chemical & Biological Engineering, Univeristy of Wisconsin
Madison, WI 53706

The scheduling of batch plants in the process industry is an important and challenging problem for both academia and practice. Despite the improvements in computer hardware and optimization software many batch scheduling problems remain hard to solve, mainly due to the structural complexity of the production facilities and the fact that the type and number of tasks (jobs) as well as the assignment of tasks to equipment units (machines) are optimization decisions.

In this work, we first show that most batch scheduling problems can, in principle, be decomposed into a master *assignment* subproblem and a slave *sequencing* subproblem. This is particularly helpful because most of the *sequencing* subproblems have been extensively studied in the OR community and can be effectively solved. We propose, therefore, a decomposition approach, where the original problem is decomposed into an aggregate master MIP subproblem and a slave subproblem that can be solved using various algorithms (e.g. Constraint Programming, heuristics, problem-specific scheduling algorithms). In the master MIP problem we determine the type and number of tasks to be carried out as well as the assignment of tasks to units. For a given assignment (i.e. fixed number and type of tasks and task-unit assignments), in the subproblem we check feasibility and solve the sequencing problem. The slave subproblem is also used to generate Benders cuts that are added in the cut-pool of the master subproblem. The proposed framework can be implemented either as an iterative scheme or as a branch-and-cut algorithm. The advantage of the proposed scheme is that it combines the complementary strengths of different solution methods. We use Mathematical Programming to optimize (i.e. identify partially good solutions), and specialized algorithms to check feasibility and solve the sequencing subproblem. The proposed decomposition approach, however, has the following two limitations:

1. It is difficult to develop "tight" formulations for the MIP master subproblem. This implies that a lot of computational time can be potentially spent for the examination of infeasible assignments.
2. It is difficult to generate strong Benders (integer) cuts, because the "source" of infeasibility cannot always be identified. The generated Benders cuts, therefore, cut-off a very small region.

To address the first limitation we develop pre-processing algorithms, while for the second one we study sub-networks of the original problem and use the solution (or the solution process) of the slave subproblem. Pre-processing algorithms are effective in two ways: a) tighten the time window of assignment constraints of the master subproblem, and b) identify infeasible assignments (that are feasible for the original master problem) and a priori generate integer cuts to exclude them. Moreover, the decomposition of the production network into subnetworks enables us to develop strong integer cuts with few integer variables. Ideally, we want to generate cuts that include only the variables that are "responsible" for an infeasible solution.

The proposed approach was implemented in three classes of problems with very good computational results (more details and preliminary results can be found in Maravelias and Grossmann (2004a, 2004b)):

1. Single-stage batch plants: The one-machine algorithm of Balas and Vazacopoulos (1995) is used for the solution of the subproblem. Pre-processing is used for the a priori generation of integer cuts. Both the minimization of makespan and the minimization of cost (with release and due times) are considered.
2. Multi-stage batch plants: The Shifting Bottleneck Procedure (SBP) of Balas et al. (1998) is used for the solution of the slave subproblem. The SBP provides us with information that can be used for the derivation of strong integer cuts. Pre-processing is also used for the generation of integer cuts.
3. Multi-purpose batch plants: The sub-problem is solved using Constraint Programming methods. A network-based pre-processing algorithm is used to tighten the MIP master problem and different types of integer cuts are developed.

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