A DIVIDE-AND-MERGE HEURISTIC FOR SINGLE MACHINE WEIGHTED TARDINESS PROBLEM

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Abstract: We introduce a divide-and-merge heuristic for the single machine weighted tardiness problem. Given an instance of the problem, our algorithm divides it into k (small) subproblem instances using a feasible solution as a basis. After optimizing each subproblem, the heuristic merges their solutions to form a complete solution to the original problem. We also provide some computational results that show the effectiveness of our heuristic.

Key words: Heuristic, Single Machine, Total Weighted Tardiness.

1. OVERVIEW

In this presentation, we introduce a divide-and-merge heuristic for the single machine minimum total weighted tardiness problem (TWTP). Suppose that we have a single machine scheduling problem with n jobs. Let p_j, d_j, and w_j for j=1,…, n be the processing time, the due date and the weight of job j, respectively. Let C_j be the completion time of job j. The tardiness of job j, T_j, is defined as Max {0, C_j - d_j}. Our objective is minimizing the total weighted tardiness, i.e. $\sum_{j=1}^{n} w_j T_j$. TWTP is represented by $1 || \sum_{j=1}^{n} w_j T_j$ within the scheduling notation of Graham et al. (1979).

One approach for solving TWTP is using exact algorithms, i.e. algorithms that are guaranteed to find the optimal solutions. Some examples of exact algorithms are dynamic programming, branch-and-bound, and mixed integer programming (MIP) based algorithms. Abdul-Razaq *et al.* (1990) provides an extensive survey on branch and bound and dynamic programming based algorithms. Since TWTP is NP-Hard (Lenstra *et al.*, 1977) real life size instances of this problem cannot be solved within a reasonable amount of computational time. For example, as discussed in Keha and Fowler (2005), there are 20 jobs instances of TWTP that cannot be solved to optimality by using MIP techniques within an hour with today's computing capabilities. Given the fact that many real world scheduling problems have more than a hundred jobs, we probably will not be able to rely only on exact algorithms to find good solutions.

Since it is not always possible to find the "optimal solution" fast enough, we use heuristics to find "good solutions" faster. Heuristics are usually polynomial time algorithms and a (hopefully) good feasible solution is obtained quickly. Currently, heuristics are the most popular algorithms to solve TWTP.

These two approaches have their own strengths: exact algorithms find the best solution while heuristics find (hopefully) good solutions faster. In this paper we use the divide-and-merge approach that combines the strengths of both approaches. Given an instance of a problem, this approach *divides* the problem into k (small) subproblem instances using a feasible solution as a basis. After optimizing each subproblem, the approach *merges* their solutions to form a complete solution to the original problem. This approach has recently been introduced by Keha and Fowler (2005).

In this presentation, we introduce the divide-and-merge heuristic. We start with an example application of the approach and then give a formal description of a generic divide-and-merge heuristic. Next, we give results of a set of experiments to test the performance of the divide-and-merge heuristic and to observe the parameters that affect the performance of the approach. We conclude with a discussion of the results and further research issues.

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