LOWER BOUNDS FOR PARALLEL MACHINE SCHEDULING

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Abstract We study the scheduling situation where a set of jobs have to be scheduled on parallel identical machines. Jobs are subjected to release dates and the objective is to minimize a regular min sum criteria $F = \sum f_i(C_i)$. In this talk, we review the existing lower bounds for this problem and we introduce two new ones. All of them are theoretically and experimentally compared.

Keywords: Machine Scheduling, Theoretical Scheduling

1. PROBLEM DEFINITION

We consider the problem where n jobs have to be scheduled on m identical parallel machines. Each job J_i is defined by a release date r_i and a processing time p_i. Preemption is not allowed and the objective is to minimize a regular min sum criteria $F = \sum f_i(C_i)$.

Although our bounds are valid for any such F, we focus, as far as experiments are concerned, on some well known criteria such as total completion time ($\sum C_i$), total weighted completion time ($\sum w_iC_i$), total tardiness ($\sum T_i$) and total weighted tardiness ($\sum w_iT_i$). Note that all four corresponding problems $Pm|r_i|\sum C_i$, $Pm|r_i|\sum w_iC_i$, $Pm|r_i|\sum T_i$ and $Pm|r_i|\sum w_iT_i$ are NP-hard in the strong sense.

2. LOWER BOUNDS

We introduce two lower bounds based on the following scheme.

- First we compute a vector $(C_{[1]}, C_{[2]}, ..., C_{[n]})$ such that $\forall i, C_{[i]}$ is a lower bound of the ith smallest completion time in any schedule.
- Second we define an assignment problem between jobs and the above completion times $C_{[1]}, C_{[2]}, ..., C_{[n]}$. The cost of assigning the job J_i to the date $C_{[u]}$ is $f_i(C_{[u]})$ and we seek to minimize the total assignment cost.

For some objective function, the assignment problem can be solved fast thanks to the problem structure. The general case is solved in cubic time as an assignment problem.

The crucial point of the above mechanism is how to compute the $C_{[i]}$ values. Our *first lower bound* computes $C_{[i]}$ as the minimum makespan of any preemptive schedule of at least i jobs in $\{J_1, ..., J_n\}$. Relying on Horn's formulation [1] for $P|r_j, pmtn|C_{max}$, we use a set of "small" MIPs to compute such $C_{[i]}$ values. For this reason, this lower bound requires a significant amount of time to be computed.

The *second lower bound*, is weaker than the first one since $C_{[i]}$ is computed as a lower bound of the minimum makespan of any preemptive schedule of at least i jobs in $\{J_1, ..., J_n\}$. Although this bound is theoretically dominated by the first one, it can be computed fast. Surprisingly, we can also use it to speed up the computation of the the first lower bound.

3. CONCLUSION

These lower bounds are compared to an LP relaxation of a time indexed MIP formulation of the problem as well as to more specific bounds, i.e., to bounds that have been proposed for one of the specific problems $Pm|r_i|\sum C_i$, $Pm|r_i|\sum w_i C_i$, $Pm|r_i|\sum T_i$ and $Pm|r_i|\sum w_i T_i$ (see for instance [2, 3]).

We have also integrated these bounds in a tree-search procedure. Preliminary results show that they allow us to drastically reduce the search space. Detailled results will be presented at the time of the conference.

References

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