

# On the Capacitated Packing-Delivering Problem with a Fixed Route

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## ABSTRACT

The packing-delivering problem is concerned with packing products with different capacity requirements (e.g., cubic feet or weight) into capacitated vehicles and scheduling the vehicle delivery operations so that a given objective is optimized while the customer demands and the vehicle capacity constraints are satisfied. In this study, we solve a variation of this problem, which involves  $K$  distinct products,  $K \geq 1$ , and a single capacitated vehicle with a fixed sequence of customer locations to be visited. The vehicle capacity is restricted by the total loading weight, and each customer may order multiple products with different quantity for each product. The objective is to minimize the total time needed to deliver to all the customers. Let  $\mathbf{P}$  denote this problem variation.

We prove that  $\mathbf{P}$  is strongly polynomial time solvable if each customer can be visited only once, but becomes NP-hard in strong sense if customers accept split deliveries. For the later case, the differences between  $\mathbf{P}$  and the known single-product split delivery problem are discussed, and the new problem properties are presented. We then analyze the conditions of optimality and show that the results from this analysis can greatly reduce the search effort without affecting the solution quality. A local optimization algorithm with a computational complexity of  $O(K \cdot n^3 \log(n))$  is proposed for the case that allows split deliveries and evaluated through 14,000 randomly generated test problems with a wide range of parameter values and product types. A consistent performance improvement achieved by the proposed local optimization algorithm over that by the optimal non-split packing-delivery schedules is observed.

Keywords: delivery scheduling, bin-packing, split delivery, fixed route

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