SCHEDULING A SINGLE MACHINE TO MINIMIZE A REGULAR OBJECTIVE FUNCTION UNDER SETUP CONSTRAINTS

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Abstract Motivated by industrial applications, we study the scheduling situation in which a set of jobs subjected to release dates and deadlines are to be performed on a single machine. The objective is to minimize a regular sum objective function $\sum_i f_i$ where $f_i(C_i)$ corresponds to the cost of the completion of job J_i at time C_i. On top of this, we also take into account setup times and setup costs between families of jobs as well as the fact that some jobs can be "unperformed" to reduce the load of the machine. We introduce lower bounds and dominance properties for this problem and we describe a Branch and Bound procedure with constraint propagation. Experimental results are reported.

Keywords: Machine Scheduling, Constraint Programming

Real manufacturing scheduling problems exhibit a number of difficult features that are often ignored in the literature. Motivated by a new testbed inspired by industrial real life situations [2], we study the one machine scheduling problem with:

- Release dates and deadlines.
- Costs dependent on the completion times of activities.
- Possibilities of leaving some jobs "unperformed."
- Setup times and costs.

In this problem, a set of jobs $\{J_1, ..., J_n\}$ subjected to release dates r_i and deadlines d_i are to be performed on a single machine. The pro-

cessing time of J_i is p_i. The objective is to minimize a regular (i.e., non-decreasing) sum objective function $\sum_i f_i$ where $f_i(C_i)$ corresponds to the cost of completing J_i at time $C_i \in [r_i + p_i, d_i]$. Two extensions of this core problem are considered:

- When the machine is overloaded some jobs can be *"unperformed"* to reduce the machine load. In such a case, an "unperformance" cost u_i is associated to each job J_i. When the job is not scheduled in its time window $[r_i, d_i]$, the cost u_i is added to the objective function. Note that if the f_i functions are null, the problem reduces to minimizing $\sum w_i U_i$, a well-known objective function in scheduling theory (see for instance, [1]).
- Due to manufacturing constraints, *setups* must be performed between jobs with different machine feature requirements. We rely on the following model: there are q families of jobs and $\phi(J_i) \in \{1, ..., q\}$ denotes the family of the job J_i. Within the same family, there is no transition time nor cost. But between consecutive jobs of families ϕ_1 and ϕ_2, at least $\delta(\phi_1, \phi_2)$ units of time must elapse. Moreover, a cost $f(\phi_1, \phi_2)$ is associated to this setup.

We describe a new lower bound for this very general problem and a Branch and Bound procedure with constraint propagation. We also introduce a set of dominance rules that reduce the search space.

To our knowledge, this is the first exact procedure for such a general problem. We tested our procedure on 88 instances of the Manufacturing Scheduling Library (MaScLib) [2], available at `www2.ilog.com/masclib`. We closed 46 of these instances. Note that when jobs can be unperformed or are subjected to setup constraints, small instances with 24 or 30 jobs are still open. We hope this will encourage other researchers to tackle the problem described in this paper.

Further work includes the development of heuristics to more efficiently explore the search space and the generalization of this procedure to more complex scheduling problems, eg, to a multi-machine environment.

References

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