# Aligning Frequencies in Cyclic Delivery Scheduling

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When facing the task of replenishing customers with stable demand rates on a long-term basis, the natural approach is to set up a cyclic distribution pattern. In this cyclic approach, customers are grouped into tours and tours are assigned to the available vehicle fleet. To ensure that a vehicle can feasibly make all tours that are assigned to it, the cycle times of these tours need to be aligned. This paper presents a new heuristic that aligns delivery frequencies of multiple tours that have to be made by a single vehicle, with the objective of minimizing cost rates. Computational experiments are set up to evaluate the performance of the new heuristic and compare it to an existing heuristic.

*Keywords:* Delivery Scheduling, Transport Scheduling, Heuristic Search.

## 1 Problem description

In many real-life distribution systems, a set of customers has to be repeatedly replenished from a depot. When these customers have stable consumption rates, the natural approach is to set up a cyclic scheme. In such a cyclic approach, customers are grouped into tours and then the best cycle time for each tour is determined. If the holding costs of the customers are taken into account, this best cycle time can be determined by an EOQ-like formula [4], that trades off transportation and holding costs. However, the problem of setting up a cyclic distribution scheme has a long-term perspective. Therefore, it is appropriate not to consider the vehicle fleet as fixed. Indeed, in the long term, the vehicle fleet is variable, such that fleet sizing needs to be taken into account. This means that the tours have to be assigned to a minimal fleet of vehicles. Campbell and Hardin [1] propose a heuristic for this problem. But, since fixed vehicle costs are usually large compared to transportation and holding costs, it may be a good idea to adjust the cycle times of one or more tours in an attempt to reduce the required number of vehicles. This article studies the problem of aligning the cycle times of tours that are assigned to a single vehicle, such that a feasible schedule can be constructed for the vehicle. Consider e.g. the two tours of Figure 1 that each take a full working day to be completed. If the optimal cycle times of these tours are two and three days respectively, then they will have to be performed on the same day once every six days. Since they each take a full day to be performed, two vehicles would be needed. However, if we align the cycle times of these tours and decide to decrease the cycle time of the second tour to two days or increase it to four days, both tours never have to be made on the same day and only one vehicle is needed. Adjusting the cycle time of the second tour to 2 days as in Figure 1 increases the variable distribution costs and decreases holding costs, thus disturbing the trade-off between these, but it helps in reducing the fixed vehicle fleet costs, so it is certainly worthwhile.

When adjusting tour cycle times, we need to know the range of feasible cycle times for each route individually. It is assumed here that no tour is ever made twice per day, so the obvious minimal cycle time for any given tour is 1 day. The maximal tour cycle time, on the other hand, is derived from the limited loading capacity of the vehicles and the limited storage capacity of the customers. With $\kappa$ the vehicle loading capacity, $S$ the subset of customers visited in the tour, $d_j$

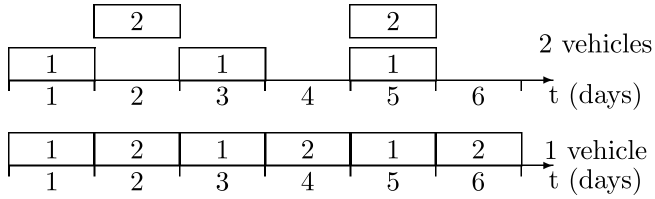



Figure 1: A simple example of cycle time alignment

the daily demand rate of customer $j$ and $\kappa_j$ the storage capacity of customer $j$, the maximal cycle time of the tour is as follows: $T_{max} = \min\left(\frac{\kappa}{\sum_{j \in S} d_j}, \min_{j \in S} \frac{\kappa_j}{d_j}\right)$.

The optimal cycle time gives the best trade-off between transportation and holding costs. Since holding costs are usually small compared to transportation costs, large delivery quantities and thus long cycle times give better trade-offs. The size of this delivery quantity is limited by the available capacities and thus the optimal cycle time is very often given by the maximal cycle time.

## Formal problem description

Given a set of $N$ different tours (indexed by $i$) with their optimal cycle times $T_{eoq,i}$ in days, their maximal cycle times $T_{max,i}$ in days and their travel times $t_i$ in hours, we need to find tour cycle times $T_i$ and a delivery schedule for a single vehicle in which tour $i$ is made once every $T_i$ days and in which the cumulative travel time of the vehicle on any given day does not exceed a given threshold (usually 8 hours). The objective is to minimize the variable distribution and holding cost rate. If no cost information is explicitly given, this objective can be replaced by the minimization of the cumulative deviation between the actual cycle times $T_i$ and the optimal cycle times $T_{eoq,i}$.

A mathematical formulation for this problem is given below. The variable $T$ in the model denotes the cycle time of the vehicle's schedule. This schedule cycle time $T$ is given by the least common multiple of all tour cycle times $T_i$. The binary variables $X_{it}$ indicate whether tour $i$ is made on day $t$ in the schedule or not.

$$\text{Min Z} = \sum_{i=1}^{N} \textbf{abs}\left(1 - \frac{T_i}{T_{eoq,i}}\right)$$

subject to:

$$\begin{aligned}
\sum_{t=1}^{T_i} X_{it} = 1 \qquad & \forall i \in 1..N \\
X_{it} = X_{i,t+T_i} \qquad & \forall i \in 1..N, t \in 1..T - T_i \\
\sum_{i=1}^{N} t_i X_{it} \leq 8 \qquad & \forall t \in 1..T \\
T_i \in \{1..T_{max,i}\} \qquad & \forall i \in 1..N \\
X_{it} \in \{0,1\} \qquad & \forall i \in 1..N, t \in 1..T
\end{aligned}$$

Because the cycle times $T_i$ (and $T$) are variables, this formulation is non-linear. When the $T_i$ and $T$ are fixed, the problem becomes a linear generalized assignment problem, which is known to be NP-hard. When the number of tours assigned to a single vehicle is limited, the problem can be solved by enumerating all possible $T_i$ combinations and solving the resulting linear MIP-model with branch-and-bound. However, this is very time-consuming. To be able to solve larger instances and to be able to evaluate a large number of solutions for the overall problem of setting up a cyclic distribution scheme with many tours and multiple vehicles, a fast heuristic is developed for the cycle time alignment (sub)problem.

## 2 Solution approach

The heuristic approach for the cycle time alignment problem consists of two iterative phases: (i) selecting tour cycle times $T_i$, starting of course from the optimal cycle times $T_{eoq,i}$ and (ii) checking the feasibility of a given $T_i$ combination by constructing a schedule. As soon as a feasible schedule is found, the procedure stops and the cycle time combination is returned.

The cycle time $T$, given by the least common multiple of the cycle times $T_i$, gives an indication of the 'alignment' of the cycle times. A high $T$ indicates that some of the tour cycle times are relative primes, meaning that the tours have to be made on the same day every now and then. To avoid having to make the tours on the same day, their cycle times have to be aligned such that they are no longer relative primes. This increases the 'alignment' and reduces the cycle time $T$.

During the alignment procedure, tours are sorted in order of increasing cycle times $T_i$, and in order of decreasing durations $t_i$ in case of a tie. Because the relative cycle time deviations are to be minimized, the idea is to adjust the larger $T_i$'s first. To this end, a critical tour is identified in the second phase, i.e. during schedule construction. In the first phase, the cycle times of all tours in the order starting from this critical tour are adjusted as follows. First, the least common multiple of all tours, which is the cycle time $T$ of these tours together, before the critical tour is calculated. Then, the cycle time of the tour is first decreased and later increased until it is a divisor or an integer multiple of $T$. Of these two, the one with smallest deviation from the tour optimal cycle time is chosen.

The second phase, where a feasible schedule has to be constructed for given cycle times $T_i$, consists of two steps. The first step is a preprocessing step that compares the $T_i$ two by two. If two tours have relative prime cycle times and cannot be made together on the same day without violating the driving time restriction, we can already conclude here that no feasible schedule exists. The tour with the higher $T_i$ then becomes the critical tour and the procedure returns to the first phase. When no conflicts are encountered in the first step, the second step is started, in which an actual schedule is constructed with a best-fit insertion heuristic, adapted from Raa et al. [2]. In this heuristic, the tours are inserted into the schedule one by one according to the given tour order, except that, if possible, no two tours with the same cycle time are inserted consecutively [2]. As soon as a tour cannot be feasibly inserted, it becomes the critical tour and the procedure returns to the first phase of cycle time alignment. If a feasible schedule is found, the problem is solved and the procedure stops.

To make this approach work, two extra rules are added. The first is that if the critical tour is the same as in the previous iteration, the tour before it in the order becomes critical. This is necessary to avoid infinite looping. The second rule looks at the vehicle utilization. If the total travel time during a cycle is very high compared to the time available in the cycle, this greatly increases the chances of resulting in infeasibilities. Therefore, when the vehicle utilization is too high, only cycle time increases are allowed in the first phase so the vehicle utilization will decrease.

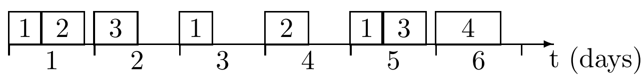
# 3 Illustrative example

| $i$ | $T_{eoq,i}$ | $T_{max,i}$ | $t_i$ |
|---|---|---|---|
| 1 | 2 | 4 | 3h |
| 2 | 3 | 4 | 4h |
| 3 | 4 | 6 | 4h |
| 4 | 7 | 7 | 6h |

Initially, the $T_i$ are equal to the optimal cycle times $T_{eoq,i}$. This gives a (very high) cycle time $T$ of 84 days. However, tours 1 and 4 conflict: they have relative prime cycle times and take more than 8 hours together. Therefore, tour 4 becomes critical and its cycle time $T_4$ is adjusted to 6 days (increasing to 8 is infeasible because $T_{max,4} = 7$), reducing $T$ to only 12 days.

With the adjusted $T_4$, no conflicts can be found in the first step of the second phase, so a schedule is constructed. This schedule construction fails when inserting tour 4, but since this was already critical, tour 3 becomes critical and its cycle time $T_3$ is adjusted to 3 days (increasing to 6 would result in a larger deviation). The cycle time $T_4$ remains unchanged because it is already aligned. This results in the feasible schedule shown below.

| $i$ | $T_i$ | $1 - \frac{T_i}{T_{eoq,i}}$ |
|---|---|---|
| 1 | 2 | 0.0% |
| 2 | 3 | 0.0% |
| 3 | 3 | 25.0% |
| 4 | 6 | 14.3% |




### Computational results

To evaluate the performance of the proposed procedure for cycle time alignment, it is tested on a set of randomly generated instances and then compared to the results obtained with the approach of Raa and Aghezzaf [3, 4]. That approach is based on relative frequencies, and iteratively determines a cycle time $T$ and a set of frequencies $k_i$ for the tours, such that the resulting tour cycle times $T/k_i$ are as close as possible to the optimal tour cycle times. The heuristic used for constructing the schedule is the same as the one used here. These computational experiments are currently being performed and results will be available soon.

# 4 Conclusion

This article presents a new heuristic solution approach for finding cycle times for tours that are assigned to a vehicle, such that the cumulative deviation from their individual optimal cycle times is minimal. The approach is very fast and effective, which is particularly useful when designing cyclic distribution patterns for replenishing large sets of customers, where many tours and vehicles are necessary and thus many alternatives need to be evaluated. Currently, computational experiments are being performed to evaluate the effectiveness of this new approach.

For future work, the proposed heuristic will need to be adapted to tackle variants of the problem with additional real-life features and constraints such as heterogenous vehicle fleets and customer time windows for delivery.

## References

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