Minimizing Makespan with Multiple Orders per Job in Mixed Flowshops

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We investigate a new scheduling problem, multiple orders per job (MOJ), to optimize the makespan in a 3-machine flowshop consisting of two types of machines: item-processing and lot-processing. MOJ problems are restricted in the number of jobs formed from orders as well as a job capacity limit. Job processing time for an item-processing machine is proportional to the size of the job, while job processing time for a lot-processing machine is not. We extend prior results by characterizing the opposing effects of item/lot machines on optimal job formation and job scheduling in flowshops that contain both types of machines. Job domination in this context is defined, and precise conditions are provided to determine when lower bounds are equivalent to smaller problems.

Keywords: Scheduling, flowshops, multiple-orders-per-job, makespan, semiconductor manufacturing.

1. Introduction

The multiple orders per job (MOJ) problem domain evolved initially from the semiconductor manufacturing industry. In this highly competitive industry, manufacturers must continually improve their processes and equipment. The shift to larger, 300-mm diameter silicon wafers increases the number of good die per wafer by almost 125%, and therefore reduces the number of wafers required to fill an individual order. Transporting these wafers within the production facility to the proper tool groups is accomplished with Front-Opening Pods (FOUPs). These transporters provide a clean atmosphere to prevent wafer surface contamination, and eliminate the risk of damage from personnel physically moving these heavy materials. Although tracking and facilty routing encourage avoiding the separation of wafers in one order, using a complete FOUP for each order is not efficient. Efficiency and equipment utilization can be improved by placing multiple orders within each transporter without splitting any order. This MOJ problem is complicated by the additional constraint of a limited number of transporters and a limited transport capacity.

The multiple-orders-per-job problem for flowshops is depicted in Figure 1. We assume a set of H orders $O=\{o_k\}$ are available for processing and scheduling on sequential machines at time zero. Each order has size s_k, representing the number of items (wafers) in the order, and P is the sum of all order sizes. To accommodate different machine speeds, the unit processing time, c_i, is dependent on machine i, but not on the order. This scenario commonly holds for semiconductor manufacturing equipment where the processing time is wafer-based. For convenience, we also define processing time ratios $\rho_i = c_{i+1}/c_i$.

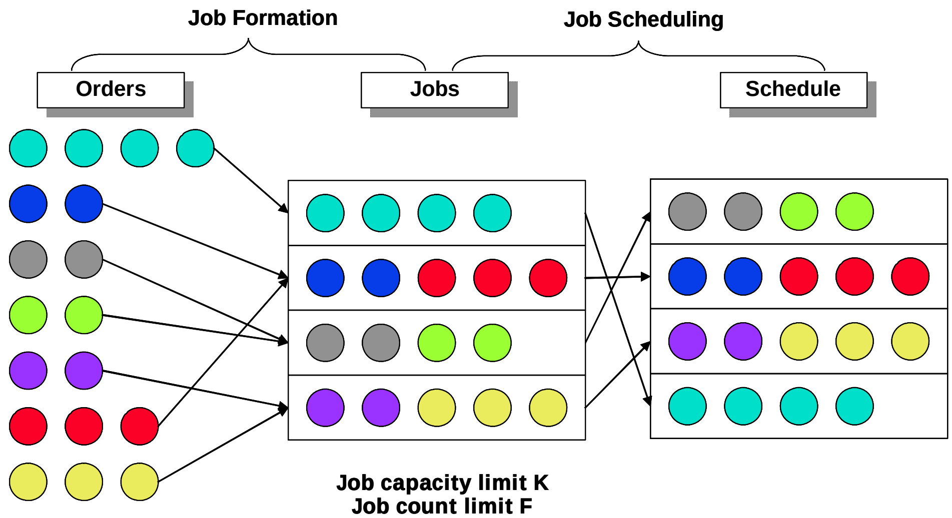


Figure 1. Multiple-Orders-per-Job Problems

Each order must be assigned to exactly one of *F* non-preemptible jobs, and multiple orders can be assigned to a single job as long as the sum of order sizes does not exceed the job size capacity *K*. If the size of an order exceeds *K*, then we assume the order is first divided into separate orders, each of size *K* or less. Each of the resulting orders is therefore wholly contained in a single job. The job processing time depends on the type of machine. On an item-processing machine, denoted "moj(item)", the job processing time is the sum of the processing times for all items in all orders in the job. On a lot-processing machine, denoted "moj(lot)", the processing time is independent of the job composition. The objective is to assign the orders to jobs and schedule the jobs to minimize the makespan for the entire set of orders.

Laub, *et al.* (2005) addressed the two-machine item-processing MOJ makespan problem by calculating a lower bound based on a relaxed problem equivalent to lot streaming with job capacity limits. The calculation of the lower bound was accompanied by equations for optimal job sizes as a function of machine unit processing times, job capacity, and the limit on the number of jobs constructed. The paper then defines an algorithm (which we reference here as "MOJF") that uses these job sizes in a modified version of the bin packing algorithm called "First or Closest Fit Decreasing" (FCFD). Orders are assigned to jobs using the optimal job sizes as "target" bin sizes. The FCFD heuristic attempts to assign orders to jobs to match the target job sizes as closely as possible, but is allowed to exceed these targets as long as the maximum job capacity *K* is not exceeded. The heuristic is quite fast, and produces solutions within a small percentage of the lower bound. Laub, *et al*. (2006 and 2006a) performed the same technique for flowshops with three item processing machines. Their work produced algorithms for precise calculation of the lower bounds as well as further heuristics, as the equations for three machines became much more complicated.

The current paper addresses the multiple orders per job problem for flowshops that contain a mix of both item processing and lot processing machines. For these mixed MOJ machine problems, we use the extended notation *Fm|moj(x,y,...)|Cmax*, where *x, y,…* represent either item-processing or lot-processing machines. For example, *F2|moj(item,lot)|Cmax* represents a two-machine flowshop where the first machine uses item processing and the second machine uses lot processing. Alternatively, for extended configurations, we use abbreviations *I* and *L* for item processing and lot processing, so that, for example, the *F4|moj(item,item,lot,lot)|Cmax* problem is abbreviated *F4|moj(IILL)|Cmax*, or simply *IILL*. Because we use corresponding lot streaming problems to determine lower bounds for the MOJ problems, we use a similar notation for these, e.g. *LS(IILL)*.

In all cases, a vector *(c_1, c_2,..., c_m)* defines the times it takes each machine to process a single item. All items in orders are considered equivalent in this work. Let P represent the total number of items in all orders, so that on an item processing machine M_i, the total processing time for all orders is Pc_i and on a lot processing machine, the total processing time is nc_i, where n is the number of jobs constructed. The required processing time for a single job on an item processing machine is x_jc_i, where x_j is the number of items in the job (the job size), and the required time for a job on a lot processing machine is simply d, regardless of the job size. In problems involving a limit on job capacity, we use K to represent the job capacity, or the maximum number of items in a job.

2. Three-Machine Mixed Flowshops

We first investigate the three-machine problem with a single lot-processing machine. The position of the lot-processing machine in a three machine flowshop is critical in its effect on the optimal job sizes.

To address this mixed-mode MOJ problem, we first investigate the optimal job sizes for the problem when K is large enough to require no capacitation for an optimal solution. Resulting equations can then be adapted for the capacitated version. The final optimal job sizes are then suitable for use as target job sizes in the First or Closest Fit Decreasing (FCFD) algorithm, and the corresponding makespan is used as the lower bound for the MOJ problems. For convenience when addressing these problems with two item-processing machines, we use d to represent the unit processing time of the lot machine, and c_1 and c_2 as the unit processing times for the two item machines, regardless of their position relative to the lot-processing machine in the flowshop. We use ρ to represent the ratio of processing times between the two item machines (c_2/c_1). As the results are dependent on whether the item processing time ratio ρ is greater than 1, less than 1, or equal to 1, these cases are presented in separate sections.

In these problems, it is not unwarranted to expect the two item machines to compete with the lot machine in terms of optimal job sizes. Potts and Baker (1989) showed that for an uncapacitated flowshop with two item processing machines, there is an optimal solution to the lot streaming problem that uses all available jobs, and is geometric in the optimal job sizes. For a flowshop with lot processing machines, however, an optimal solution will use the minimum number of jobs possible. For the mixed cases, the question is when and how does the speed of the lot processing machine affect the geometric pattern of optimal job sizes for the item machines.

We say that *the item-processing machines dominate the lot processing machine* if the optimal solution to a mixed 3-machine MOJ problem is equivalent, in terms of job sizes, to the corresponding 2-machine problem instance without the lot processing machine. We say that *the lot-processing machine dominates the item-processing machines* if the optimal solution to the problem is equivalent, in terms of job sizes, to the corresponding single-machine problem without the item-processing machines, i.e., all orders are placed in a single job.

We expect that the item processing machines would dominate the lot processing machine for small values of d relative to c_1 and c_2, and the lot-processing machine would dominate the item-processing machines for large values of d. This is, in fact, the case, and the following theorems define these intervals of values for d precisely for each of the three configurations, IIL, LII, and ILI.

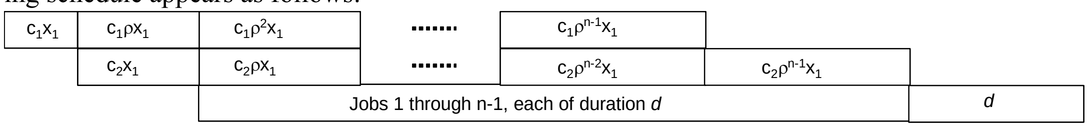
2.1. Uncapacitated IIL

For the following theorems, we relax the problem by assuming that the total amount of processing material is infinitely divisible. The condition for the lot machine dominating the item machines does not depend on which item machine is faster. Theorem 1 states the case for a dominating lot machine when machine M_1 is faster than M_2, but the result for the opposite relationship is the same.

Theorem 1 (Lot Machine Dominance). For the 3-machine lot streaming problem where machines M_1 and M_2 are item-processing machines and machine M_3 is a lot-processing machine, let n be the maximum number of jobs in which to distribute P units of required processing. Let $\rho=c_2/c_1>1$ and $a_z=P(c_1+c_2)$. Then $d \geq a_z$ if and only if the optimal job configuration is to place all processing in one job. Furthermore, the makespan is $P(c_1+c_2)+d$.

Corollary 1: For any $F3 \mid moj(mixed) \mid Cmax$ problem with two item-processing machines and one lot-processing machine, the lot machine dominates the item machines whenever $d \geq P(c_1+c_2)$, where the c_i are the unit processing times of the two item machines, regardless of their positions relative to the lot machine.

On the other hand, to characterize the conditions for item machine dominance, we must consider which item machine is the faster one. For the case when $\rho>1$, the optimal item machine dominating schedule appears as follows:



Theorem 2 (Item Machine Dominance for IIL and $\rho>1$). For the uncapacitated 3-machine MOJ problem where machines M_1 and M_2 are item-processing machines and machine M_3 is a lot-processing machine, let n be the maximum number of jobs in which to distribute P units of required processing. Let $\rho=c_2/c_1>1$ and $a_1 = \dfrac{Pc_2\rho\left(\rho^{n-1}-1\right)}{(n-1)\left(\rho^n-1\right)}$. Then the item-processing machines dominate the lot-processing machine if and only if $d \leq a_1$. Furthermore, the makespan is $c_1 P \dfrac{\rho-1}{\rho^n-1} + c_2 P + d$.

We can now characterize the precise values of d that bound the dominating configurations for $\rho>1$, as depicted below.

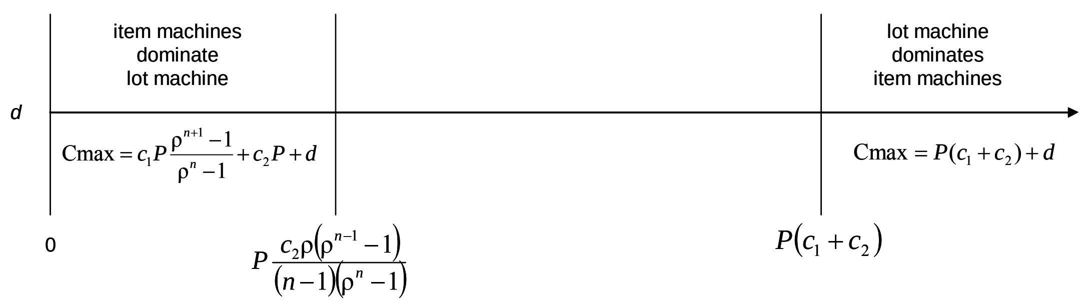
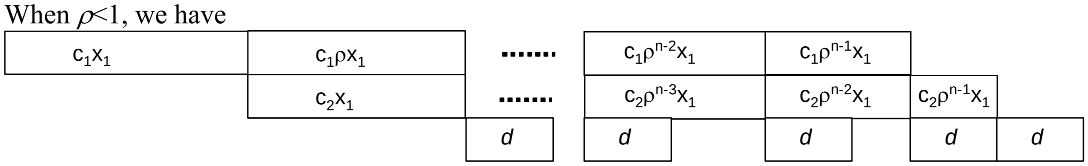


Figure 2. Machine Dominance for IIL, $\rho>1$

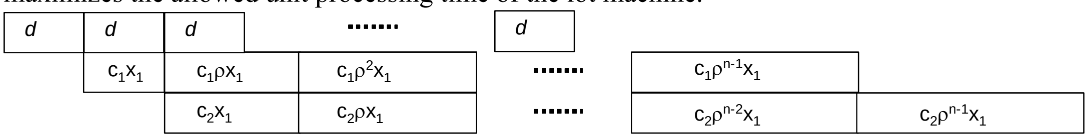
When $\rho<1$, we have



Theorem 3 (Item Machine Dominance for IIL and $\rho<1$). For the uncapacitated 3-machine MOJ problem where machines M_1 and M_2 are item-processing machines and machine M_3 is a lot-processing machine, let n be the maximum number of jobs in which to distribute P units of required processing. Let $\rho=c_2/c_1<1$ and $a_1 = c_2\rho^{n-1}P\frac{\rho-1}{\rho^n-1}$. Then the item-processing machines dominate the lot-processing machine if and only if $d \le a_1$. Furthermore, the makespan is $c_1P\frac{\rho-1}{\rho^n-1}+c_2P+d$ (The case for $\rho=1$ is addressed in Section 2.4).

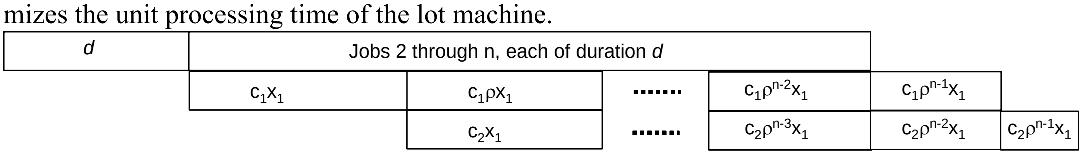
2.2. Uncapacitated LII

When $\rho > 1$ and the item machines are dominant, we have the following optimal schedule that maximizes the allowed unit processing time of the lot machine.



Theorem 4 (Item Machine Dominance for LII and $\rho>1$). For the uncapacitated 3-machine MOJ problem where machines M_2 and M_3 are item-processing machines and machine M_1 is a lot-processing machine, let n be the maximum number of jobs in which to distribute P units of required processing. Let $\rho=c_2/c_1>1$ and $a_1 = c_1P\frac{\rho-1}{\rho^n-1}$. Then the item-processing machines dominate the lot-processing machine if and only if $d \le a_1$. Furthermore, the makespan is $d+c_1P\frac{\rho-1}{\rho^n-1}+c_2P$.

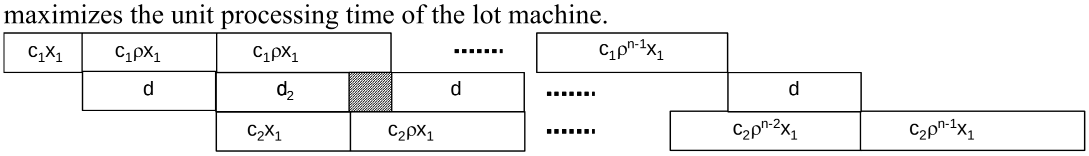
When $\rho < 1$ and item machines are dominant, we have the following optimal schedule that maximizes the unit processing time of the lot machine.



Theorem 5 (Item Machine Dominance for LII and $\rho<1$). For the uncapacitated 3-machine MOJ problem where machines M_2 and M_3 are item-processing machines and machine M_1 is a lot-processing machine, let n be the maximum number of jobs in which to distribute P units of required processing. Let $\rho=c_2/c_1<1$ and $a_1 = \frac{Pc_1\left(\rho^{n-1}-1\right)}{(n-1)\left(\rho^n-1\right)}$. Then the item-processing machines dominate the lot-processing machine if and only if $d \le a_1$. Furthermore, the makespan is $d+c_1P\frac{\rho-1}{\rho^n-1}+c_2P$.

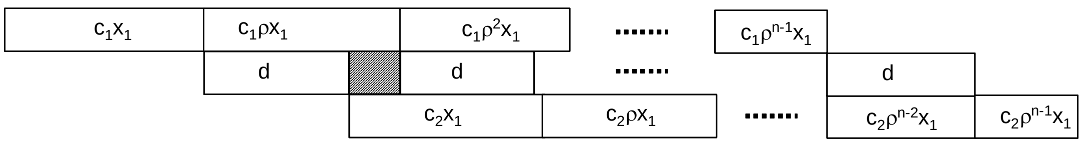
2.3 Uncapacitated ILI

When $\rho > 1$ and the item machines are dominant, we have the following optimal schedule that maximizes the unit processing time of the lot machine.



Theorem 6 (Item Machine Dominance for ILI and $\rho>1$). For the uncapacitated 3-machine MOJ problem where machines M_1 and M_3 are item-processing machines and machine M_2 is a lot-processing machine, let n be the maximum number of jobs in which to distribute P units of required processing. Let $\rho=c_2/c_1>1$. Then the item-processing machines dominate the lot-processing machine if and only if $d \le c_2 P \frac{\rho - 1}{\rho^n - 1}$. Furthermore, the makespan for any such schedule is $c_1 P \frac{\rho - 1}{\rho^n - 1} + d + c_2 P$.

When $\rho < 1$ and item machines are dominant, we have the following optimal schedule that maximizes the unit processing time of the lot machine.



Theorem 7 (Item Machine Dominance for ILI and $\rho<1$). For the uncapacitated 3-machine MOJ problem where machines M_1 and M_3 are item-processing machines and machine M_2 is a lot-processing machine, let n be the maximum number of jobs in which to distribute P units of required processing. Let $\rho=c_2/c_1<1$. Then the item-processing machines dominate the lot-processing machine if and only if $d \le c_2 \rho^{n-2} P \frac{\rho - 1}{\rho^n - 1}$. Furthermore, the makespan for any such schedule is $c_1 P \frac{\rho - 1}{\rho^n - 1} + d + c_2 P$.

2.4 Three Machine Summary

All configurations for three machines with two item machines have been addressed except for ρ=1. When the speeds of the item machines are equal, then the optimal sizes of all jobs are equal. Regardless of the position of the lot machine, the item machines are dominant when the unit processing time of the lot machine does not exceed the duration of any job executing on an item machine, i.e., c_2P/F.

Theorem 8 (Item Machine Dominance $\rho=1$). When the two item machines operate at the same speed, i.e., $\rho=1$, then the optimal job size for all jobs is P/n. Hence, the item machines dominate if and only if $d \le c_2 P / n$

The following tables summarize the results for mixed-mode processing with three machines.

Table 1: Conditions for Two Item Machines Dominating a Lot Machine

	ρ < 1	ρ = 1	ρ > 1
IIL	$d \le c_2 \rho^{n-1} P \dfrac{\rho - 1}{\rho^n - 1}$	$d \le c_2 P / n$	$d \le \dfrac{P c_2 \rho (\rho^{n-1} - 1)}{(n-1)(\rho^n - 1)}$
ILI	$d \le c_2 \rho^{n-2} P \dfrac{\rho - 1}{\rho^n - 1}$	$d \le c_2 P / n$	$d \le c_2 P \dfrac{\rho - 1}{\rho^n - 1}$
LII	$d \le \dfrac{P c_1 (\rho^{n-1} - 1)}{(n-1)(\rho^n - 1)}$	$d \le c_2 P / n$	$d \le c_1 P \dfrac{\rho - 1}{\rho^n - 1}$

<u>Corollary 2</u>. The optimal makespan for any item-machine dominating 3-machine flowshop with one lot machine is $c_1 P \dfrac{\rho - 1}{\rho^n - 1} + c_2 P + d$.

Figure 3 plots the maximum unit processing time for the lot machine to ensure item machine dominance as a function of ρ, by varying c_2 while holding other values constant: c_1=10, P=100, n=6.

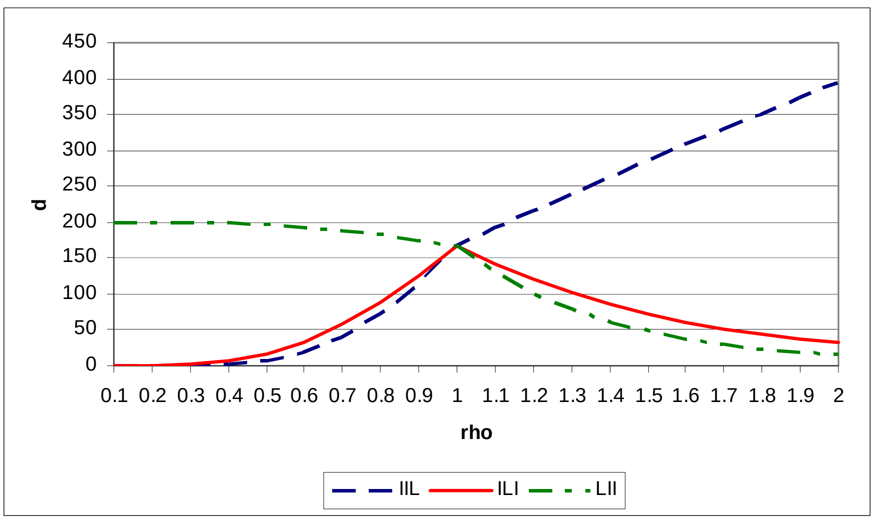


Figure 3. Maximum unit processing time for a lot machine under item-machine dominance

Given a set of two item machines and one lot machine where ρ<1, the bound d that determines whether the item machines dominate increases if the lot machine is moved back towards the beginning of the flowshop, i.e., $d_{IIL} < d_{ILI} < d_{LII}$. We claim that the makespan for any item-machine dominating configuration is at least a good as that for a configuration of the same machines that is not item-dominating. Hence, if there is a choice in placement of the lot machine, it should be placed upstream when ρ<1 (and downstream when ρ>1).

In Figure 4, the number of available jobs has been increased from 6 to 25. As the number of jobs increases, the conditions for item machine dominance become much more restrictive when ρ<1 in an IIL configuration, and when ρ>1 in an LII configuration.

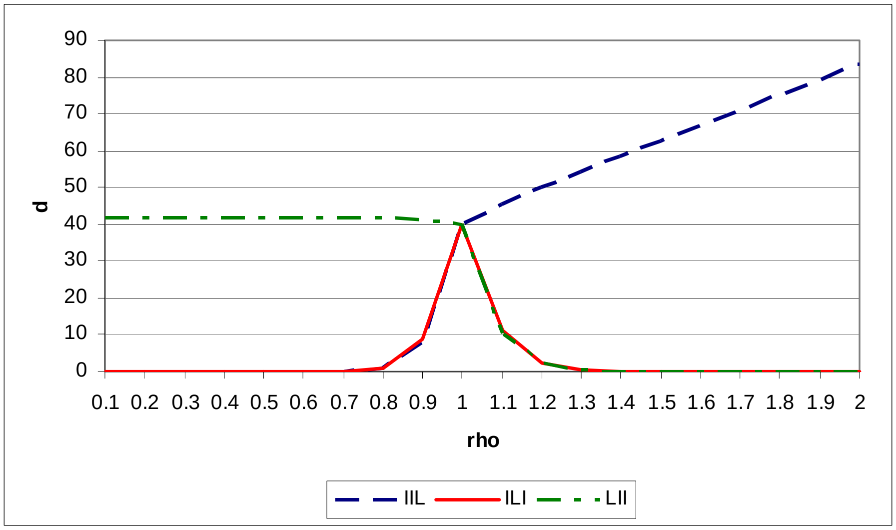


Figure 4. Item Machine Dominance Conditions for 25 Jobs

3. Conclusions

Optimizing the makespan for the multiple-orders-per-job problem requires both job formation and job scheduling, while adhering to restrictions of limited job capacity and limited number of jobs allowed, without allocating any order across multiple jobs. In a multiple-orders-per-job problem, each machine is either item-processing, where the job processing time is proportional to the total number of items in all orders within a job, or lot-processing, where the job processing time is not dependent on job size. Prior results have provided heuristics for flowshops of two and three machines, where each machine is item-processing. In this paper, we examine three-machine mixed flowshops, where there is a mix of both item-processing machines and lot-processing machines. We provide formulas for the conditions under which the item-processing machines dominate the lot-processing machine, and vice versa. Under machine domination, the optimal job sizes, assuming infinite order divisibility, are equivalent to smaller problems previously solved. This results in simple calculation of lower bounds for the optimal solution, as well as target job sizes for previously defined heuristics. We also provide a complete characterization of the relationships between the lot-processing speed and the ratio of the item-processing speeds under machine domination conditions. In future work, the authors will present heuristics and experimental results for three machine problems and discuss extensions of our approach for use in mixed flowshops with more than three machines.

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