

Optimization of the Departure Schedule at a Public Transit Terminal with Multiple Destinations

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The paper considers the schedule optimization problem for public transit networks. In particular, we are interested in optimizing the departure schedule for the lines that leave from a transit terminal, in which passengers are supposed to arrive, according to a given schedule, and split between different lines of a service, or even change mode of transportation in case of intermodal systems. The aim is to decide the schedule for the output lines, in such a way to find the optimal trade-off between the sum of service operative costs and passengers waiting times at the transit terminal. We present a model that is able to represent the problem in presence of a single destination for all the passengers, and its extension to the general case of multiple destination. We show similarities between this models and those proposed in literature for other well known combinatorial optimization problems, also drawing some conclusions about the complexity of the proposed models.

Keywords: Location, Logistics, Schedule Synchronization, Transit Network, Transport Scheduling.

1 Introduction

Intermodal transportation is based on the combined use of different modes of transport to move passengers from their own origins to destinations. The efficiency of such systems depends on the possibility to ensure transfers involving switching passengers from one route to another with limited waiting times. A crucial role in the efficiency of an intermodal system is played by transfer or transit terminals, i.e. locations where users can split line and/or modes of transport during their trips. The central issue is often the maximal synchronization of the schedule, given a set of transit lines crossing each other in some point of the networks, in order to minimize passenger waiting times. Such a problem is widely treated in literature as the Schedule Synchronization Problem (SSP). Klemmt and Stemme [14] and Domschke [9] considered the problem of schedule synchronization for public transit networks with the objective of minimizing the sum of users transfer waiting times assuming given operation hours. Keudel [13] proposed an interactive optimization tool aiming at reducing the total users waiting times and the number of users exceeding a given time limit. Desilets and Rousseau [7] presented some results for the application of a variant of the SSP on the network of Montreal. In [21], Voss formulated the SSP as a multicommodity network design problem, exploiting the quadratic semi-assignment problem, and proposed a tabu search algorithm to solve the problem. Daduna and Voss [6] provided the application of the tabu search algorithm for the SSP on some case studies of practical interest. In [1], Adamski formulated and solved the problem of synchronizing different transit lines with shared route segments. Ceder et al. [3, 4] addressed the problem of generating the timetable for a given network of buses so as to minimize their synchronization. Wong and Leung proposed in [20] a mixed integer programming formulation aiming at the reduction of the waiting times for the passengers of a railway system, also proposing a heuristic approach. Schroder and Solchenbach [17] proposed a quadratic semi-assignment model based on offset decision variables and a system of assignment of qualitative penalties to the solutions. Most of the proposed models are in general characterized by a complexity such that the size of the

instances solvable at the optimum within reasonable computational times is limited and often not compatible with the dimension of real applications. In this paper we are interested in the dimensioning of the public transit service, deciding how many output lines should be activated, given certain levels of demand, in such a way to balance operative costs and user costs. In the following, we refer to this as the Schedule Optimization Problem (SOP). Hence, we propose a mathematical model based on a time-space representation of the network to describe this problem, and possible extensions in order to take into account the presence of different hypothesis. The similarity with other known combinatorial problems makes it possible to deduce some conclusions about the complexity of the proposed models.

The paper is organized as follows. In Section 2 the general description of the SOP is defined and illustrated. In Section 3 a mathematical model for the single destination SOP is proposed with the indication of the computational complexity. In Section 4 the extension to the multiple destination case is provided. Finally, some conclusions are presented.

2 The schedule optimization problem for a transit terminal

A transfer or transit terminal is a location, within an intermodal transportation system, where users can split lines and/or modes of transportation during their own origin-destination trips. Within a time horizon T , we consider the presence of a given set of input lines I , i.e. transit lines starting from a set of origins and arriving at the transit terminal, and a set of output lines O , i.e. transit lines that start from the terminal towards a set of given destinations. We associate to each input line:

- the arrival time at the transit terminal;
- a line-destination demand representing the number of passengers yielded by this line and directed to each destination;

Each of the output lines is characterized by:

- a destination;
- the time of departure from the transit terminal;
- a capacity, i.e. the maximum number of passengers that can be simultaneously transported on the line;
- an activation cost, paid by the service operator to activate an output line.

A passenger, during his origin-destination trip, will choose an input line $i \in I$ and will arrive at time a_i at the terminal; afterwards he will choose the first available output line $k \in O$ to reach the destination. Being $b_k \in T$ the departure time of such line from the terminal, each passenger using this trip will have a waiting time equal to $b_k - a_i$. The Schedule Optimization Problem (SOP) consists in determining the number of lines to be activated, and the departure time for each of these lines, in order to optimize a certain performance index, satisfying at the same time all the transport demand present at the transit terminal. We assume, as a performance index to be minimized, the weighted sum of the total waiting times for the users (user cost) and of the total activation costs of the lines (operator cost). An appropriate calibration of the weights used in the objective function permits to find the desired tradeoff between user and operator costs.

We propose a formulation for the SOP based on a discrete time expansion of the terminal node. The time horizon T is divided into n time periods, obtained by dividing T by the length of a time

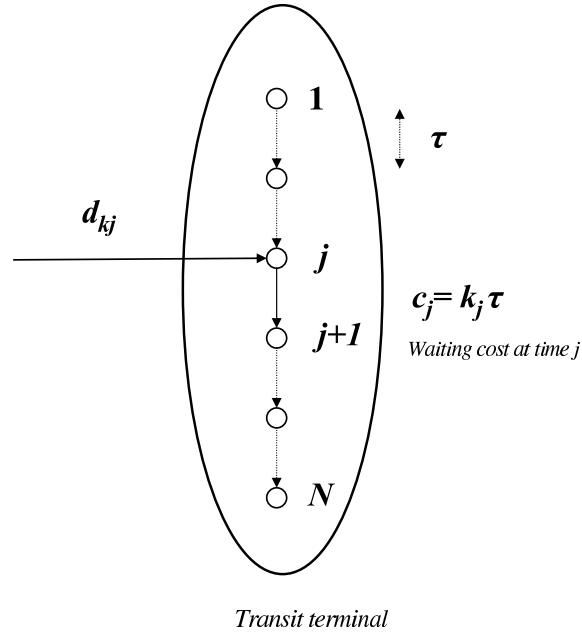


Figure 1: Sketch of the time-expanded transit terminal node: each holdover arc $(j, j+1)$ is associated with a waiting cost c_j .

unit τ . Therefore, the time expansion of the terminal node over the time horizon T is given by the graph $G_T = (N_T; A_T)$, being:

- $N_T = \{1, \dots, n\}$
- $A_T = \{(j, j+1) : \forall j \in \{1, \dots, n-1\}\}$

In this way, each node j in the time-expanded terminal node corresponds to a time instant of the time horizon T , that can coincide with the arrival and/or departure time of some line. Each node j is linked only to the successive node $j+1$; we refer to each arc $(j, j+1) \in A_T$ as holdover arc. To each holdover arc $(j, j+1) \in A_T$ is associated the waiting time τ . We can now define the waiting cost $c_j = k_j \cdot \tau$, with k_j representing a set of possible weights associated to the waiting time at time j . To each node $j \in N_T$ is associated a set of weights $d_{k,j}$ representing the total number of passengers that are supposed to arrive at the terminal at time j , and directed to the destination k . Figure 1 illustrates the discrete time expansion of the terminal node.

3 A mathematical model for the single destination problem

In the single destination SOP we suppose that the output lines are directed to a single common destination. Hence, it is possible to formulate the single destination SOP by defining the following sets of variables (see also Figure 2):

- $x_j, j \in N_T$, as continuous variables indicating the number of passengers which remain in the terminal during the time interval $[j, j+1]$, waiting for an available departure line;
- $q_j, j \in N_T$ as the number of passengers leaving the terminal at time j toward the destination using an output line activated at time j ;

- $y_j, j \in N_T$ as binary variables assuming value 1 only if an output line is activated at time j .

In this way, the model is given by:

$$\min \quad \sum_{j \in N_T} c_j \cdot x_j + \sum_{j \in N_T} f_j \cdot y_j \quad (1)$$

$$\text{s.t.} \quad x_j = x_{j-1} + d_j - q_j \quad \forall j \in N_T \quad (2)$$

$$q_j \leq M \cdot y_j \quad \forall j \in N_T \quad (3)$$

$$x_1 = x_{n+1} = 0 \quad (4)$$

$$x_j \geq 0, \quad q_j \geq 0, \quad y_j \in \{0, 1\}$$

being:

- d_j the sum of passengers arriving at the terminal at time j and directed to the common destination;
- c_j the waiting cost associated to a passenger waiting at the transit terminal during interval $[j, j + 1]$;
- f_j the fixed cost paid by the operator for the activation of a line starting at time j from the terminal.

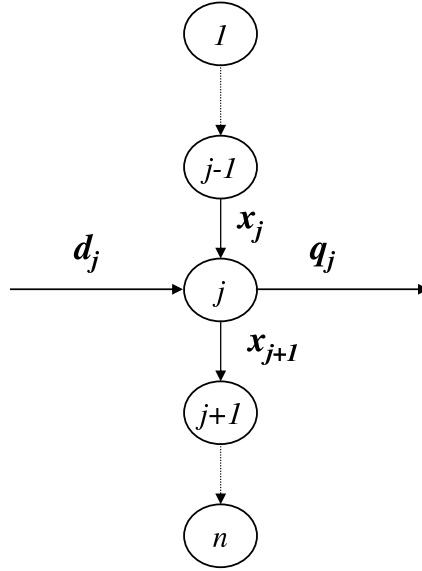


Figure 2: A scheme of the set of variables in the formulation.

This model uses $2 \cdot n$ continuous variables and n binary variables. The objective function (1) consists of the sum of the total users waiting costs and the total activation costs. Constraints (2) express the conservation of passenger flows at each time instant, while conditions (4) ensure that the number of passengers which stay in the terminal at the beginning and at the end of the time horizon is equal to 0. Constraints (3) ensure that variable q_j is equal to 0 if there is no output line starting at time j ($y_j = 0$). If we suppose that each of the activated output lines is large enough to

contain all the passengers that, at the time of the line departure, are waiting in the transit terminal (assuming that $M \geq \sum_{j=1}^i d_j$, $\forall i \in N_T$ in constraints (3), we refer to this case as the uncapacitated single destination SOP, otherwise we can consider a capacity upper bound Q_j for the number of passengers on a transit line which departs at time j , and replace constraints (3) by

$$q_j \leq Q_j \cdot y_j \quad \forall j \in N_T \quad (5)$$

In this case we refer to the model as the capacitated single destination SOP.

3.1 Complexity of this model

In order to define the complexity we show that the model becomes perfectly equivalent to a very common formulation of the well known single-item Lot Sizing Problem (see for instance [8, 12, 19]). If we slightly modify the model 1-4 by replacing d_j with $-d'_j$ and q_j with $-q'_j$, we can view d'_j as the item quantity, or demand, that must be ready in a manufacturing system at time j , q'_j as the quantity of items to be produced at each time period, and x_j as the inventory of items at time j . In particular, it is possible to show that the uncapacitated single destination SOP falls in the class of polynomially solvable problems. This can be proved, for instance, by recalling and adapting the proof for the dynamical programming algorithm proposed in [18] by Wagner and Within. To do that, it suffices to prove that the following property holds:

- (i) There exists an optimal solution with $x_j \cdot q_j = 0$, $\forall j \in N_T$ (whenever a line is activated, all the passengers get on board, and the terminal remains empty).
- (ii) There exists an optimal solution in which, if $x_j > 0$, then $x_j = D_{kj}$, $k \leq j$, being $D_{kj} = \sum_{i=k}^j d_i$ (if someone leaves from the terminal at time j , the amount of passengers leaving at time j is equal to the entire demand of the $k + 1$ consecutive periods from $j - k$ to j).

Proof. Let x_j and q_j be concurrently strictly positive for some $j \in N_T$. It would be feasible to increase q_j by x_j units and set x_j to zero, reducing the value of the objective function by $c_j \cdot x_j$. Therefore, at the optimum $x_j \cdot q_j = 0$, $\forall j \in N_T$. The second statement follows immediately.

The introduction of capacity constraints (5) makes the problem *NP*-hard. Indeed, it now corresponds to the Capacitated Lot Sizing model, that was proved in [10] to be *NP*-hard by reduction from the Knapsack Problem.

4 A mathematical model for the multi-destination problem

The extension of model (1)-(4) to the more general case with multiple destinations can be performed through a modification of the notation. The presence of a set K of possible destinations, can be taken into account by introducing the following variables:

- x_{kj} , $k \in K$, $j \in N_T$, as continuous variables representing the number of passengers which remain in the terminal during the time interval $[j, j + 1]$, being directed to destination k ;
- q_{kj} , $k \in K$, $j \in N_T$, as the number of passengers leaving the terminal at time j toward the destination k ;
- y_{kj} , $k \in K$, $j \in N_T$, as binary variables which indicate whether an output line is activated at time j with destination k .

A scheme of this notation is sketched in Figure 3. The model for the multi-destination SOP can now be stated as follows:

$$\min \quad \sum_{k \in K} \sum_{j \in N_T} (c_j \cdot x_{kj} + f_j \cdot y_{kj}) \quad (6)$$

$$\text{s.t.} \quad x_{kj} = x_{k,j-1} + d_{kj} - q_{kj} \quad \forall k \in K, \forall j \in N_T \quad (7)$$

$$q_{kj} \leq Q \cdot y_{kj} \quad \forall k \in K, \forall j \in N_T \quad (8)$$

$$\sum_{k \in K} q_{kj} \leq C_j \quad \forall j \in N_T \quad (9)$$

$$x_{k1} = x_{k,n+1} = 0 \quad \forall k \in K \quad (10)$$

$$x_{kj} \geq 0, \quad q_{kj} \geq 0, \quad y_{kj} \in \{0, 1\}$$

with c_j and Q having the same meaning of the single destination problem, and C_j representing the maximum number of passengers leaving the terminal at time j due to the capacity of the terminal. The objective (6) holds the same meaning of the single destination case, and represents the sum of total users waiting costs and the total activation costs. Constraints (7) represent the conservation of flow passengers directed to the same destination k at each time j . Constraints (8) are the capacity constraints associated to the output lines, that guarantee at the same time the activation of the related binary variable. Restrictions (9) limit the total number of passengers leaving the terminal at time j , while constraints (10) ensure that the number of passengers in the terminal at the beginning and at the end of the time horizon is equal to 0.

4.1 Complexity of the models

Adopting the same procedure shown for the single destination case, if we replace d_{kj} with $-d'_{kj}$, and q_{kj} with $-q'_{kj}$, we can interpret d'_{kj} as the external demand for an item k at time j , and q'_{kj} as the production quantity for item k at time j . In this way, the model (6)-(10) can be viewed as the well known multi-item Capacitated Lot Sizing problem which is known to be NP-Hard [2],[10].

In absence of restrictions (9) it would be possible to decompose the multi-destination problem into $|K|$ independent capacitated single destination problems, which we showed to be NP-Hard.

5 Conclusion

In this paper we defined the Schedule Optimization Problem at a transit terminal which consists of the determining the number of the lines and the departure times for each of these lines, directed to a set of possible destinations, in order to minimize the sum of the total users waiting costs and of the total activation costs of the lines. For this problem we provided some integer linear mathematical programming for the single destination and the multi-destination case. We showed that these problems are reducible respectively to the single item and the multi-item Lot Sizing problems, which are known to be NP-Hard in presence of capacity constraints. The analogy with these very well known optimization problems, let us consider the possibility to solve the SOP by using some of the exact and approximated methods proposed in literature for the Lot Size models. Further direction of research can rely on the possibility of extensions of the model to schedule lines connecting more transit terminals.

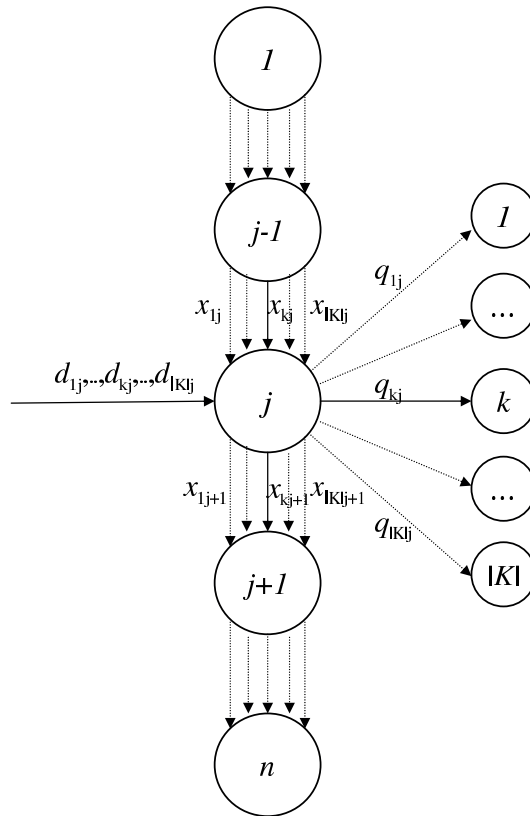


Figure 3: A scheme of the set of variables in the multi-destination model.

References

- [1] A. Adamski (1995), Transfer optimization in public transport, in: J.R. Daduna et al. (eds.), Computer – Aided transit scheduling. Proceedings of the 6th international workshop on computer – aided scheduling of public transport, Lisbon, Portugal, July 6 – 9, 1993. Berlin: Springer Verlag, *Lect. Notes Econ. Math. Syst.* 430, 23 – 38.
- [2] G.R. Bitram, H.H. Yanasse (1982), Computational complexity of the capacitated lot size problem, *Management Science* **28**, 1174 – 1186.
- [3] A. Ceder, B. Golany, O. Tal (2001), Creating bus timetables with maximal synchronization, *Transportation Research Part A* **35**, 913 - 928.
- [4] A. Ceder, O. Tal (1999), Timetable Synchronization for Buses, in: Wilson NHM (ed), Computer – Aided Transit Scheduling, *Lecture Notes in Economics and Mathematical Systems* **471**, Springer, Berlin, 245 – 258.
- [5] J.R. Daduna, J.M.P. Paixao (1995), Vehicle Scheduling for Public Mass Transit, in: J.R. Daduna, I. Branco, J.M.P. Paixao (eds), Computer – Aided Scheduling of Public Transport, *Lecture Notes in Economics and Mathematical Systems* **430**, Springer, Berlin, 76 – 90.

- [6] J.R. Daduna, S. Voss (1995), Practical Experiences in Schedule Synchronization, in: J.R. Daduna, I. Branco, J.M.P. Paixao (eds), *Computer – Aided Scheduling of Public Transport, Lecture Notes in Economics and Mathematical Systems* **430**, Springer, Berlin, 39 – 55.
- [7] A. Desilets, J.M. Rousseau (1992), SYNCRO: A Computer – Assisted Tool for the Synchronization of Transfers in Public Transit Networks, in: M. Desrochers, J.M. Rousseau(eds), *Computer – Aided Scheduling of Public Transport*, Springer - Verlag, 153 – 166.
- [8] A. Drexl, A. Kimms (1997), Lot sizing and scheduling — survey and extensions, *European Journal of Operational Research* **99**(2), 221 – 235.
- [9] W. Domschke (1989), Schedule synchronization for public transit networks, *OR Spektrum* **11**, 17 – 24.
- [10] M. Florian, J.K. Lenstra, A.H.G. Rinnooy Kan (1980), Deterministic production planning algorithms and complexity, *Management Science* **26**(7), 669 – 679.
- [11] C. Fleurent, R. Lessard, L. Seguin (2004), Transit Timetable Synchronization: Evaluation and Optimization, *Proceedings of the 9th International Conference on Computer – Aided Scheduling of Public Transport*, San Diego CA (U.S.A.), August 9-11.
- [12] B. Karimi, S.M.T. Fatemi Ghomi, J.M. Wilson (2003), The capacitated lot sizing problem: a review of models and algorithms, *Omega* **31**(55) (Oxford), Elsevier, 365 – 378.
- [13] W. Keudel (1988), Computer – aided line network design (DIANA) and minimization of transfer times in networks (FABIAN), in: Daduna, J.R., Wren, A., (eds), *Computer – Aided Transit Scheduling, Lecture Notes in Economics and Mathematical Systems* **308**, 315 – 326, Springer, Berlin.
- [14] W.D. Klemmt, W. Stemme (1988), Schedule synchronization for public transit networks, in: Daduna, J.R., Wren, A., (eds), *Computer – Aided Transit Scheduling, Lecture Notes in Economics and Mathematical Systems* **308**, 327 – 335, Springer, Berlin.
- [15] J. Krarup, P.M. Pruzan (1983), Simple plant location problem: Survey and synthesis, *European Journal of Operation Research* **12**(11), 36 – 81.
- [16] C.S. Revelle, G. Laporte (1996), The Plant Location Problem: New Models and Research Prospects, *Operations Research* **44**(6), 864 – 874.
- [17] M. Schroder, I. Solchenbach (2006), Optimization of transfer quality in regional public transit, Technical Report, Fraunhofer ITWM, 29 S.
- [18] H.M. Wagner, T.M. Within (1958), Dynamic version of the economic lot size model, *Management Science* **5**, 89 – 96.
- [19] L.A. Wolsey (1995), Progress with single – item lot – sizing, *European Journal of Operational Research* **86**, 395 – 401.
- [20] R.C.W. Wong, J.M.Y. Leung (2004), Timetable synchronization for mass transit railway, *9th International Conference on Computer - Aided Scheduling of Public Transport*, San Diego, California, August 9 - 11.
- [21] S. Voss (1992), Network design formulation in schedule synchronization, in: Desrochers M., Rousseau J.M. (eds), *Computer - Aided Scheduling of Public Transport*, Springer – Verlag, 137 – 152.