Scheduling Single Round Robin Tournaments with Fixed Venues

Rafael A. Melo
Department of Computer Science, Universidade Federal Fluminense, Rua Passo da Pátria 156, Niterói, RJ 24210-240, Brazil, melo@ic.uff.br
Sebastián Urrutia
Department of Computer Science, Universidade Federal de Minas Gerais, Av. Antônio Carlos 6627, Belo Horizonte, MG 31270-010, Brazil, surrutia@dcc.ufmg.br
Celso C. Ribeiro
Department of Computer Science, Universidade Federal Fluminense, Rua Passo da Pátria 156, Niterói, RJ 24210-240, Brazil, celso@inf.puc-rio.br

Sports scheduling is a very attractive application area due to the importance of the problems in practice and to their interesting mathematical structure. We introduce a new problem with practical applications, consisting in scheduling a compact single round-robin tournament with fixed venue assignments for each game. Two integer programming formulations are proposed and compared. Comparative numerical results are presented.

Keywords: Sports scheduling, timetabling, integer programming.

1 Problem statement

The field of sports scheduling has been attracting the attention of an increasing number of researchers in multidisciplinary areas such as operations research, scheduling theory, constraint programming, graph theory, combinatorial optimization, and applied mathematics. Particular importance is given to round robin scheduling problems in which each team is associated with a particular venue, due to their relevance in practice and to their interesting mathematical structure. The difficulty of the problems in the field leads to the use of a number of approaches, including integer programming [10, 15], constraint programming [6], hybrid methods [3], and heuristic techniques [1, 14]. We refer to [4, 11] for literature surveys.

In round robin tournaments, every team face each other a fixed number of times in a given number of rounds. Every team face each other exactly once in a single round robin (SRR) and twice in a double round robin (DRR). If the number of rounds is minimum and every team plays exactly one game in every round, then the tournament is said to be compact. Each team has its own venue at its home city. Each game is played at the venue of one of the two teams in confrontation. The team that plays at its own venue is called the home team and is said to play a home game, while the other is called the away team and is said to play an away game. A schedule must not only determine in which round each game will be played, but also at which venue.

The problem of scheduling round robin tournaments is often divided into two subproblems. The construction of the timetable consists in determining the round in which each game will be played. The home-away pattern (HAP) set determines in which condition (home or away) each team plays in each round. The timetable and the home-away pattern set determine the tournament schedule.

Some round robin scheduling problems consider both the construction of the timetable and of the home-away pattern set. As an example, the traveling tournament problem (TTP) [5] calls for a schedule minimizing the total distance traveled by the teams.

However, either the timetable or the HAP set of the schedule may be fixed and known beforehand in some situations. In the first case, the timetable is given and the problem consists in finding a feasible HAP set optimizing a certain objective function. The break minimization problem [13] deals with the minimization of the number of breaks (two consecutive home games or two consecutive away games of the same team) in the schedule, while the timetable constrained distance minimization problem [12] calls for the minimization of the total distance traveled.

In the second case, the HAP set is predetermined and a timetable is requested. There is a feasibility issue, since not every HAP set may be associated with a compatible timetable. A necessary condition for feasibility is given in [9]. The problem of constructing a timetable compatible with a given HAP set optimizing certain objective appears as a subproblem in several approaches to solve real-life scheduling problems, see e.g. [10].

A Home-Away Assignment (HAA) [8] is an assignment of a venue to each game. A HAA is balanced if the difference between the number of home games and the number of away games is at most one for every team. If the timetable is fixed, i.e., the round of every game is known, the HAP set and the HAA give the same information and determine the same schedule.

We consider and formulate the problem of scheduling single round robin tournaments with fixed home-away assignments. The venue of each game is known beforehand and the problem consists in determining a timetable minimizing a certain objective function. Variants of this problem find interesting applications in real-life leagues whose DRR tournaments are divided into two SRR phases. Games in the second phase are exactly the same of the first phase, except for the inversion of their venues. Therefore, the venues of the games in the second phase are known beforehand and constrained by those of the games in the first phase. This is the case e.g. of the Chilean soccer professional league [2] and of the German table tennis federation of Lower Saxony [7].

We assume that each team has its own venue at its home city. All teams are initially at their home cities, to where they return after their last away game. The distance $d_{ij} \geq 0$ from the home city of team i to that of team j is known beforehand. A road trip is a sequence of consecutive away games played by a team at the venues of its opponents. This team travels from the venue of one opponent to that of the next, without returning home.

Let G be a HAA, whose elements are ordered pairs of teams. The game between teams i and j is represented either by the ordered pair (i, j) or by the ordered pair (j, i). In the first case, the game between i and j takes place at the venue of team i; otherwise, at that of team j. Consequently, for every two teams i and j, either $(i, j) \in G$ or $(j, i) \in G$.

Given an even number n of teams and a home-away assignment G, the Traveling Tournament Problem with Fixed Venues (TTPFV) consists in finding a compact single round robin schedule compatible with G, such that the total distance traveled by the teams is minimized and no team plays more than three consecutive home games or three consecutive away games. Section 2 describes two integer programming formulations of this problem, with respectively $O(n^3)$ and $O(n^5)$ variables. The strength of their linear relaxations is compared in Section 3. Computational results are presented in Section 4. Concluding remarks are drawn in the last section.

2 Integer programming formulations

2.1 Formulation with $O(n^3)$ variables

We define the following decision variables:

$$z_{tjk} = \begin{cases} 1, & \text{if team } t \text{ plays at home against team } j \text{ in round } k, \\ 0, & \text{otherwise;} \end{cases}$$

$$y_{tij} = \begin{cases} 1, & \text{if team } t \text{ travels from the facility of team } i \text{ to the facility of team } j, \\ 0, & \text{otherwise.} \end{cases}$$

The above variables are used in the formulation (1)-(12) of TTPFV, with $O(n^3)$ variables:

$$\min F_1(z,y) = \sum_{t=1}^{n}\sum_{i=1}^{n}\sum_{j=1}^{n} d_{ij} y_{tij} \tag{1}$$

subject to:

$$\sum_{q=1}^{n-1} z_{tjq} = 1, \qquad \forall\,(t,j) \in G \tag{2}$$

$$\sum_{\substack{j=1\\ j\neq t}}^{n} (z_{tjk} + z_{jtk}) = 1, \qquad t = 1,\ldots,n, \quad k = 1,\ldots,n-1 \tag{3}$$

$$y_{tij} \geq z_{it,k-1} + z_{jtk} - 1, \qquad t,i,j = 1,\ldots,n \text{ with } t \neq i \neq j, \quad k = 2,\ldots,n-1 \tag{4}$$

$$y_{tit} \geq z_{it,k-1} + \sum_{\substack{j=1\\ j\neq t}}^{n} z_{tjk} - 1, \qquad t,i = 1,\ldots,n \text{ with } t \neq i, \quad k = 2,\ldots,n-1 \tag{5}$$

$$y_{tti} \geq \sum_{\substack{j=1\\ j\neq t}}^{n} z_{tj,k-1} + z_{itk} - 1, \qquad t,i = 1,\ldots,n \text{ with } t \neq i, \quad k = 2,\ldots,n-1 \tag{6}$$

$$y_{tti} \geq z_{it1}, \qquad t,i = 1,\ldots,n \text{ with } t \neq i \tag{7}$$

$$y_{tit} \geq z_{it,n-1}, \qquad t,i = 1,\ldots,n \text{ with } t \neq i \tag{8}$$

$$\sum_{q=k}^{k+3}\sum_{\substack{j=1\\ j\neq t}}^{n} z_{jtk} \leq 3, \qquad t = 1,\ldots,n, \quad k = 1,\ldots,n-4 \tag{9}$$

$$\sum_{q=k}^{k+3}\sum_{\substack{j=1\\ j\neq t}}^{n} z_{jtk} \geq 1, \qquad t = 1,\ldots,n, \quad k = 1,\ldots,n-4 \tag{10}$$

$$z_{tjk} \in \{0,1\}, \qquad t,j = 1,\ldots,n, \quad k = 1,\ldots,n-1 \tag{11}$$

$$0 \leq y_{tij} \leq 1, \qquad t,i,j = 1,\ldots,n. \tag{12}$$

The objective function (1) defines the minimization of the total distance traveled by the teams. Constraints (2) ensure that each game occurs exactly once. Constraints (3) guarantee that each team plays one game in each round. Constraints (4) enforce team t to perform a trip from the home city of team i to that of team j if it plays two consecutive away games against teams i and j, in this order. Constraints (5) enforce team t to perform a trip from the home city of team i to its own home city if it has an away game against the latter followed by a home game in the next round. Constraints (6) enforce team t to travel from its own home city to that of team i to play away against the later after a home game in the previous round. Constraints (7) enforce team t to travel to the home city of team i if it plays away against the latter in the first round. Constraints (8) enforce team t to return from the home city of team i if it plays away against the latter in the last round. Constraints (9) establish that team t cannot play more than three consecutive away games, while constraints (10) guarantee that team t cannot play more than three consecutive home games. Constraints (11) define the integrality requirements for the z variables. The y variables act as generalized upper bounds to constraints (4) to (8). Since their costs in the objective function to be minimized are non-negative, they will always assume the minimal possible value, which is necessarily either 0 or 1. Therefore, the integrality requirements on the y variables may be replaced by lower and upper bounds expressed as constraints (12).

This formulation has $O(n^4)$ constraints: $O(n^2)$ of types (2), (3), (7), (8), (9) and (10), $O(n^3)$ of types (5) and (6), and $O(n^4)$ of type (4).

2.2 Formulation with $O(n^5)$ variables

This formulation considers complete road trips. The variables represent road trips of different sizes, giving a more direct representation of the problem. Three new decision variables are defined and used in the third formulation:

$$w^1_{tik} = \begin{cases} 1, & \text{if team } t \text{ starts, in round } k\text{, a road trip visiting team } i \text{ and} \\ & \quad \text{returning home in round } k+1 \text{ (with } t \neq i) \\ 0, & \text{otherwise;} \end{cases}$$

$$w^2_{tijk} = \begin{cases} 1, & \text{if team } t \text{ starts, in round } k\text{, a road trip visiting first team } i\text{, then team } j, \\ & \quad \text{and returning home in round } k+2 \text{ (with } t \neq i \neq j) \\ 0, & \text{otherwise;} \end{cases}$$

$$w^3_{tijlk} = \begin{cases} 1, & \text{if team } t \text{ starts, in round } k\text{, a road trip visiting first team } i\text{, then team } j, \\ & \quad \text{next team } l\text{, and returning home in round } k+3 \text{ (with } t \neq i \neq j \neq l) \\ 0, & \text{otherwise.} \end{cases}$$

Two dummy rounds (indexed by -1 and 0) are created to simplify the formulation. The variables corresponding to every road trip starting in any of these fictitious rounds is set to 0. The auxiliary costs c_{ij}, c_{ijm}, and c_{ijml} represent the costs of road trips of length one, two, and three performed by team i, respectively:

$$c_{ij} = d_{ij} + d_{ji}, \tag{13}$$

$$c_{ijm} = d_{ij} + d_{jm} + d_{mi}, \tag{14}$$

$$c_{ijml} = d_{ij} + d_{jm} + d_{ml} + d_{li}. \tag{15}$$

The new variables are used to reformulate TTPFV as model (16)-(21) below, with $O(n^5)$ variables. Although the number of variables increases with respect to the previous formulation, we notice that the number of constraints is quite smaller.

$$\min F_3(w^1, w^2, w^3) \quad = \quad \sum_{k=1}^{n-1} \sum_{i=1}^{n} \sum_{\substack{j=1 \\ (j,i)\in G}}^{n} [c_{ij} w^1_{ijk} \; + \; \sum_{\substack{m=1 \\ (m,i)\in G}}^{n} (c_{ijm} w^2_{ijmk} \; + \; \sum_{\substack{l=1 \\ (l,i)\in G}}^{n} c_{ijml} w^3_{ijmlk})] \tag{16}$$

subject to:

$$\sum_{i=1}^{n} \sum_{j=1}^{n} [\sum_{k\in\{-1,0\}} w^1_{ijk} + \sum_{m=1}^{n} (\sum_{k\in\{-1,0,n-1\}} w^2_{ijmk} + \sum_{l=1}^{n} \sum_{k\in\{-1,0,n-2,n-1\}} w^3_{ijmlk})] = 0 \tag{17}$$

$$\sum_{k=1}^{n-1} \{w^1_{ijk} + \sum_{\substack{m=1 \\ (m,i)\in G}}^{n} [(w^2_{ijmk} + w^2_{imjk}) + \sum_{\substack{l=1 \\ (l,i)\in G}}^{n} (w^3_{ijmlk} + w^3_{imjlk} + w^3_{imljk})]\} = 1, \qquad \forall (j,i) \in G \tag{18}$$

$$\sum_{\substack{j=1\\(j,i)\in G}}^{n}\{w^1_{ijk}+\sum_{\substack{m=1\\(m,i)\in G}}^{n}[(w^2_{ijmk}+w^2_{ijm,k-1})+\sum_{\substack{l=1\\(l,i)\in G}}^{n}(w^3_{ijmlk}+w^3_{ijml,k-1}+w^3_{ijml,k-2})]\}+$$
$$+\sum_{\substack{j=1\\(i,j)\in G}}^{n}\{w^1_{jik}+\sum_{\substack{m=1\\(m,j)\in G}}^{n}[(w^2_{jimk}+w^2_{jmi,k-1})+\sum_{\substack{l=1\\(l,j)\in G}}^{n}(w^3_{jimlk}+w^3_{jmil,k-1}+w^3_{jmli,k-2})]\}=1,$$
$$i=1,\ldots,n,\quad k=1,\ldots,n-1 \quad (19)$$

$$\sum_{\substack{j=1\\(j,i)\in G}}^{n}\{w^1_{ijk}+w^1_{ij,k+1}+\sum_{\substack{m=1\\(m,i)\in G}}^{n}[w^2_{ijm,k-1}+w^2_{ijmk}+w^2_{ijm,k+1}+$$
$$+\sum_{\substack{l=1\\(l,i)\in G}}^{n}(w^3_{ijml,k-2}+w^3_{ijml,k-1}+w^3_{ijmlk}+w^3_{ijml,k+1})]\}\le 1,\qquad i=1,\ldots,n,\quad k=1,\ldots,n-2 \quad (20)$$

$$\sum_{q=k}^{k+3}\{\sum_{\substack{j=1\\(j,i)\in G}}^{n}[w^1_{ijq}+\sum_{\substack{m=1\\(m,i)\in G}}^{n}((w^2_{ijmq}+w^2_{ijm,q-1})+\sum_{\substack{l=1\\(l,i)\in G}}^{n}(w^3_{ijmlq}+w^3_{ijml,q-1}+w^3_{ijml,q-2}))]\}\ge 1,$$
$$i=1,\ldots,n,\quad k=1,\ldots,n-4 \quad (21)$$

$$w^1_{ijk}\in\{0,1\},\qquad i,j=1,\ldots,n,\text{ with } i\neq j,\quad k=-1,\ldots,n-1 \quad (22)$$
$$w^2_{ijmk}\in\{0,1\},\qquad i,j,m=1,\ldots,n,\text{ with } i\neq j\neq m,\quad k=-1,\ldots,n-1 \quad (23)$$
$$w^3_{ijmlk}\in\{0,1\},\qquad i,j,m,l=1,\ldots,n,\text{ with } i\neq j\neq m\neq l,\quad k=-1,\ldots,n-1. \quad (24)$$

The objective function (16) minimizes the total traveled distance. Constraints (17) set to 0 the variables associated to road trips starting at the dummy rounds -1 and 0, to road trips of size two and three starting at the last round and those of size three starting at round $n-2$. Constraints (18) ensure each game occurs exactly once. They represent the fact that each game $(j,i)\in G$ should be played in a road trip of team i formed by one, two or three away games. Constraints (19) enforce that team i is either playing an away game or another team is visiting it in each round. This is achieved by setting to one the sum of all road trips of team i which include round k with the sum of all road trips of other teams which visit i in round k. Constraints (20) forbid team i to be engaged in simultaneous or consecutive (i.e., without returning to its home city) road trips in round k. Constraints (21) state that team i must be outside its home city to play away at least once along every four consecutive rounds. Constraints (22)-(24) guarantee the integrality requirements.

This formulation has $O(n^2)$ constraints of types (18) to (21).

3 Linear relaxation bounds

We refer to the formulations with $O(n^3)$ and $O(n^5)$ variables, respectively, as models N3 and N5. We denote by $\overline{\text{N3}}$, and $\overline{\text{N5}}$, respectively, their linear relaxations. Let $LB3$ and $LB5$ be, respectively, the linear relaxation bounds provided by formulations N3 and N5.

Theorem 1: $LB5 \geq LB3$.
Sketch of the proof: Let $(w^{1^*}, w^{2^*}, w^{3^*})$ be an optimal solution to model $\overline{\text{N5}}$ satisfying constraints (18)-(21). We build a solution $(\hat{y}, \hat{z})$ to model $\overline{\text{N3}}$ by defining:

$$\hat{z}_{tik} = w^{1^*}_{itk} + \sum_{\substack{l=1 \\ (l,i)\in G}}^{n} [(w^{2^*}_{itlk} + w^{2^*}_{ilt,k-1}) + \sum_{\substack{m=1 \\ (m,i)\in G}}^{n} (w^{3^*}_{itlmk} + w^{3^*}_{iltm,k-1} + w^{3^*}_{ilmt,k-2})],$$
$$t, i = 1, \dots, n, \quad k = 1, \dots, n-1 \qquad (25)$$

$$\hat{y}_{tij} = \sum_{k=1}^{n-1} [w^{2^*}_{tijk} + \sum_{\substack{l=1 \\ (l,t)\in G}}^{n} (w^{3^*}_{tijlk} + w^{3^*}_{tlij,k})], \qquad t, i, j = 1, \dots, n, \text{ with } t \neq i \neq j \qquad (26)$$

$$\hat{y}_{ttj} = \sum_{k=1}^{n-1} \{w^{1^*}_{tjk} + \sum_{\substack{l=1 \\ (l,t)\in G}}^{n} [w^{2^*}_{tjlk} + \sum_{\substack{m=1 \\ (m,t)\in G}}^{n} w^{3^*}_{tjlmk}]\}, \qquad t, j = 1, \dots, n, \text{ with } t \neq j \qquad (27)$$

$$\hat{y}_{tit} = \sum_{k=1}^{n-1} \{w^{1^*}_{tik} + \sum_{\substack{l=1 \\ (l,t)\in G}}^{n} [w^{2^*}_{tlik} + \sum_{\substack{m=1 \\ (m,t)\in G}}^{n} w^{3^*}_{tlmik}]\}, \qquad t, i = 1, \dots, n, \text{ with } t \neq i. \qquad (28)$$

It can be proved by variable substitution that $(\hat{y}, \hat{z})$ satisfies constraints (2)-(12) (i.e., it is feasible to $\overline{\text{N3}}$) and that $F_3(w^{1^*}, w^{2^*}, w^{3^*}) = F_1(\hat{y}, \hat{z})$. The result follows, since $LB5 = F_3(w^{1^*}, w^{2^*}, w^{3^*}) = F_1(\hat{y}, \hat{z}) \geq F_1(y^*, z^*) = LB3$, where (y^*, z^*) is an optimal solution to $\overline{\text{N3}}$. □

4 Computational results

To evaluate and compare the proposed integer programming models, we created 20 home-away assignments for each of the national league (nl) TTP instances in [16] with up to 12 teams. Ten instances in each group have their home-away assignments randomly determined. The other ten instances correspond to balanced home-away assignments: the venues of all games were randomly selected and the iterative method in Knust and Thaden [8] was used to balance the home-away assignments.

The models have been solved by CPLEX 10.0, using the API Concert Technology compiled with g++ 4.0.2. The computational experiments were performed on a Pentium D machine with a 3.0 GHz processor and 2 Gbytes of RAM memory, running under version 2.6.12 of Linux.

The linear relaxations of the two formulations described in Section 2 were solved for all instances. The results are summarized in Table 1. For each value of n, the second and third columns of this table give, respectively, the average lower bounds provided by models N3 and N5 for the feasible instances. The two last columns give the average and maximum gaps between the bounds $LB3$ and $LB5$, i.e., by how much $LB5$ improves upon $LB3$. The average bounds $LB5$ provided by model N5 are far better than those generated by the other formulation. As an example, the average bound $LB5$ for $n = 12$ improves by more than ten times the average bound $LB3$.

We also compared the formulations in terms of the solver capability to find optimal integral solutions. We run CPLEX for at most two hours to solve each instance using each formulation. The results are shown in Table 2. For each number of teams, we give the number of feasible instances, the number of instances solved to optimality using models N3 and N5, the average gap between $LB5$ and the optimal value F_3^*, and the average and maximum times to find an optimal solution

Table 1: Linear relaxation lower bounds

n	avg($LB3$)	avg($LB5$)	avg(($LB5 - LB3$)/$LB3$) (%)	max(($LB5 - LB3$)/$LB3$) (%)
4	2064.41	5162.60	150.08	191.59
6	3290.97	13979.0	331.50	396.69
8	4007.89	21477.0	458.58	550.88
10	3762.23	32504.80	764.71	809.42
12	5667.44	58194.74	935.20	1017.92

using model N5. The two models were able to find the optimal solutions to all small instances with $n \leq 6$. For $n = 8$, CPLEX was able to solve only five instances to optimality with model N3, but it was able to find the optimal solutions to all 20 instances with model N5. CPLEX was not able to solve instances with $n > 8$. The quality of the lower bounds $LB5$ (on average at 7.7% from optimality) helped the solver to find optimal solutions in less than one minute of computation time for most instances with $n = 8$.

Table 2: Integral solutions

n	Feasible	opt(N3)	opt(N5)	avg(($F_3^* - LB5$)/F_3^*) (%)	avg. time (s)	max. time (s)
4	20	20	20	0.0	< 0.1	< 0.1
6	17	17	17	2.4	0.2	0.7
8	20	5	20	7.7	25.6	61.8

5 Conclusion

In this paper, we introduced the traveling tournament scheduling problem with fixed venues.Two integer programming formulations for the problem were proposed. The linear relaxation of the formulation with $O(n^5)$ variables provides the strongest bounds and improves the ability of integer programming solvers to find optimal solutions.

The quality of the lower bounds $LB5$ (on average at 7.7% from optimality) helped the solver to find optimal solutions in less than one minute of computation time for all but one instance with $n = 8$. All instances with eight teams were solved in small computation times, but none with 10 teams could be solved in two hours of processing time. This result is similar to those observed for the TTP, which can be easily solved for $n = 6$, but is very hard to solve for $n = 8$.

We are currently investigating the problem of minimizing the number of breaks with fixed venues. Preliminary work leads to conjecture that the minimal number of breaks with balanced fixed home-away assignments is always smaller than or equal to the number of teams in the tournament.

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