Round-robin Sports Scheduling from a Graph Colouring Perspective
A Case Study in Rugby Union Scheduling

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1 Introduction

In this study we examine the problem of producing sports leagues that are “round-robin” in structure. Such structures occur when we have $n$ teams (or competitors) in a league, and each team is required to play all other teams exactly $m$ times within a fixed number of rounds. Examples include the Six Nations Rugby Championship (where $n = 6$ and $m = 1$), the England and Wales County Cricket Championship ($n = 9$, $m = 2$), and most European and South American national soccer leagues. In practice the most common league structures are single and double round-robins, where $m = 1$ and 2 respectively.

It has been known for many years [8] that valid round-robin schedules using the minimal $m(n-1)$ rounds can be constructed for any positive integer $n$. It is also known that the number of distinct round-robin schedules grows exponentially with $n$, since this figure is monotonically related to the number of non-isomorphic one-factorisations of a complete graph with $n$ vertices (which is also known to increase exponentially with $n$). Such a growth rate implies that, for non-trivial problems, examining all possible schedules in order to identify the one that best fits the needs of the user will not always be possible in reasonable time (in practice, “non-trivial” values for $n$ would appear to be anything above 10 when $m = 1$ [4].)

Recent work on round-robin scheduling problems has tended to focus on the use of metaheuristic-based approaches, which have shown to be effective for exploring the large search spaces associated with such problems [1,3,10,12]. The strategy used in such methods is to apply a two-stage scheme, whereby a valid round-robin schedule is first constructed, with changes then being made to the schedule via the application of neighbourhood operators that preserve this validity. Until now, such approaches have tended to focus on a specific formulation of round-robin scheduling problem, known as the Travelling Tournament Problem, originally proposed by Easton [5].

In this work we have chosen to look at the problem of round-robin scheduling in more general terms. Specifically, we choose to view it as a type of graph colouring prob-
Graph colouring is a classical combinatorial optimisation problem for which much research has been conducted, both from the perspective of solving graph colouring problems [2,6,7,9] and in exploring the spaces of feasible and/or optimally coloured graphs [9,11,13]. By using such principles, in this work we analyse the ability of a number of well-known graph colouring algorithms for solving round-robin scheduling problems of different sizes. We also show how the graph colouring model can be extended in order to cope with additional hard (i.e. mandatory) constraints that can often be imposed in practical situations. Finally, we also examine a number of neighbourhood operators, originating from the graph colouring literature, that can then be utilised in order to explore the space of feasible round-robin schedules.

In the final part of this work we consider a complex real world scheduling problem that was given to us by the Welsh Rugby Union, based at the Millennium Stadium in Cardiff, Wales. This problem involves producing a double round-robin tournament using \(2(n - 1)\) rounds and features a number of additional hard constraints to do with stadium sharing, stadium availability, and pre-assignment issues. In addition, soft constraints involving the spread of related matches throughout the tournament are also imposed. We propose two separate algorithms for tackling this problem, both that make use of our proposed algorithmic operators, and we demonstrate their ability to produce solutions that are superior to those currently used by the league, in short periods of time.

References