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Master Physician Scheduling Problem

Aldy Gunawan • Hoong Chuin Lau

Abstract In this paper, we study a real-world problem arising from the operations of a hospital service provider, which we term the master physician scheduling problem. It is a tactical planning problem of assigning physician duties to the defined time slots/shifts over a time horizon incorporating a large number of constraints and complex physician preferences. The goals are to satisfy as many physicians' preferences and duty requirements as possible while ensuring optimum usage of available resources. We propose mathematical programming models that represent different variants of this problem. The developed models were tested on a real case from the Surgery Department of a large local government hospital, as well as on randomly generated problem instances. The computational results for solving the model are reported together with some comparison and analysis of the optimal solutions obtained. We show that this approach can significantly reduce the time and effort required to construct the master physician scheduling schedule.

Keywords: master physician scheduling and rostering problem, mathematical programming, optimization, preferences.

1 Introduction

Recently, there has been a greatly increased interest in hospital operations management which involves optimized assignment of individuals including physicians, nurses and administrators. One of the complex tasks in hospital management is to design a physician schedule, which requires taking into account a large number of constraints and preferences.

A physician schedule provides an assignment of physicians to perform different duties in the hospital timetable. Unlike nurse rostering problems which have been extensively studied in the literature (e.g. Ernst et al, 2004; Beliën and Demeulemeester, 2005; Petrovic and Berghe, 2008), in physician scheduling, maximizing satisfaction only matters, as physician retention is the most critical issue faced by hospital administrations (Carter and Lapierre, 2001). In addition, while nurse schedules must adhere to collective union agreements, physician schedules are more driven by personal preferences. In general, planning the schedules for

Aldy Gunawan
Research Fellow, The Logistics Institute – Asia Pacific, National University of Singapore
7 Engineering Drive 1, Singapore 117574
E-mail: tliag@nus.edu.sg

Hoong Chuin Lau
Associate Professor, School of Information Systems, Singapore Management University
80 Stamford Road, Singapore 178902
E-mail: hclau@smu.edu.sg

physicians requires satisfying a very large number of (often conflicting) constraints and preferences. Carter and Lapierre (2001) also provides the fundamental differences between physicians and nurses scheduling problems.

To our knowledge, research on physician scheduling focuses primarily on a single type of duty, such as the emergency room (e.g. Vassilacopoulos, 1985; Beaulieu et al., 2000; Carter and Lapierre, 2001; Gendreau et al., 2007), the operating room (e.g. Testi et al., 2007; Burke and Riise, 2008), the physiotherapy and rehabilitation services (Ogulata et al., 2008).

In this paper, we consider the problem of generating a master schedule for the physicians within a hospital service by taking a full range of day-to-day duties/activities of the physicians (including surgery, clinics, scopes, calls, administration) into consideration. Our problem, termed the **Master Physician Scheduling Problem**, is the tactical planning problem of assigning physician activities to the time slots over a time horizon incorporating a large number of rostering and resource constraints together with complex physician preferences. The goals are to satisfy as many physicians' preferences and duty requirements as possible while ensuring optimum usage of available resources such as clinics and operating theatres.

The major contributions/highlights of this paper are as follows:

(1) We take a physician-centric approach to solving this problem, since physician retention is the most critical issue faced by hospital administrations worldwide.

(2) Using mathematical models, we provide a comprehensive empirical understanding of the tradeoff of constraints and preferences against resource capacities.

The paper is organized as follows. Section 2 provides a review of the literature. Section 3 gives a detailed description of the problem. The mathematical programming models are presented in Section 4. Our developed models are tested on a real case from the Surgery Department of a large local government hospital, as well as on randomly generated problem instances. Computational results are reported together with our analysis in Section 5. Finally, we provide some concluding perspectives and directions for future research in Section 6.

2 Literature Review

Personnel scheduling and rostering is the process of constructing optimized work timetables for staff in an organization, and is becoming a critical concern in service organizations such as hospitals, hotels and airlines. A number of reviews in personnel scheduling and rostering research have appeared in Aggarwal (1982), Burke et al. (2004), Ernst et al. (2004). A categorization of comprehensive and representative solution techniques employed for different rostering problems are found in Ernst et al. (2004).

Beaulieu et al. (2000) claimed that their work was the first to present a mathematical programming approach for scheduling physicians in the emergency room. The proposed model is based on multi-objective integer programming theory. The constraints are partitioned into four different categories according to the types of rules to which they correspond: compulsory constraints, ergonomic constraints, distribution constraints, and goal constraints.

Gendreau et al. (2007) presented several generic forms of the constraints encountered in six different hospitals in the Montréal area (Canada) as well as several possible solution techniques for solving the problem. The constraints of the physician scheduling problem can be classified into four categories: supply and demand, workload, fairness and ergonomic constraints. In this paper, we are concerned about the physicians' preferences instead of fairness constraints.

A number of exact and heuristic algorithms for various scheduling problems encountered in hospitals were summarized by Beliën (2007). Beaulieu et al. (2000) proposed a mixed 0-1 programming formulation of the physician scheduling problem. The objective function is the sum of deviation (penalty) of some constraints. The problem was then solved by a heuristic approach based on a partial branch-and-bound. Buzon and Lapierre (1999) applied Tabu Search to the acyclic schedules. The cost of the solution is the sum of the costs of all physician schedules, which each one represents the sum of all penalties associated with the unsatisfied constraints.

Rousseau et al. (2002) and Bourdais et al. (2003) applied constraint programming to the physician scheduling problem. This method has been applied to the nurse scheduling problem (Bard and Purnomo, 2005). The solution technique can also be applied to the physician scheduling problem after some minor modifications.

3 Problem Description

The problem addressed in this study is to assign different physician duties (or activities) to the defined time slots over a time horizon incorporating a large number of constraints and complex physician preferences. In this paper, we assume the time horizon to be one work week (Mon-Fri), further partitioned into 5 days and 2 shifts (AM and PM).

Physicians have a fixed set of duties to perform, and they may specify their respective *ideal schedule* in terms of the duties they like to perform on their preferred days and shifts, as well as shifts-off or days off. Taking these preferences together with resource capacity and rostering constraints into consideration, our goal is to generate an *actual schedule*. As shown in Figure 1 as example, the ideal schedules might not be fully satisfied in the actual schedule. That may occur in two scenarios:

- Some duties have to be scheduled on different shifts or days – which we term *non-ideal scheduled duties* (e.g. Physician 2 Tuesday duties).
- Some duties simply cannot be scheduled due to resource constraints – which we term *unscheduled duties* (e.g. Physician 1 Friday PM duty).

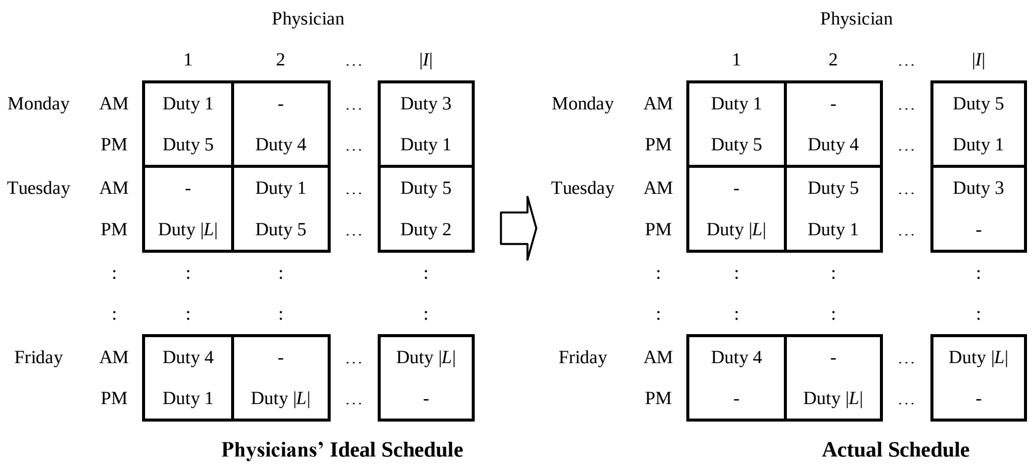

Figure 1. Example of Master Physician Scheduling Problem

Although each hospital has its unique rostering requirements, the following summarizes some common requirements treated in this paper:

- No physician can perform more than one duty in any shift.
- The number of resources (e.g. operating theatres, clinics) needed cannot exceed their respective capacities at any time. For simplicity, we assume that each type of activity does not share its resources with another type of activities – for example, operating theatres and clinics are used to perform surgery and out-patient duties respectively.
- Ergonomic constraints: Some duties are regarded as *heavy* duties, such as surgery and endoscopy duties. The following ergonomic constraints hold:
  - If a physician is assigned to a heavy duty in the morning shift, then he cannot be assigned to another type of heavy duty in the afternoon shift *on the same day*. However, it is possible to assign *the same* type of heavy duties in consecutive shifts on the same day.

- Similarly, a physician cannot also be assigned to another type of heavy duty in the morning shift on a particular day if he has been assigned to a heavy duty in the afternoon shift on the previous day.

- The number of activities allocated to each physician cannot exceed his contractual commitments, and do not conflict with his external commitments. In this paper, we assume external commitments take the form of physicians' request for shifts-off or days-off, and hence no duty should be assigned to these requests.

In this paper, we study different problem settings. The basic problem is to minimize the total number of unscheduled duties without any physician preferences nor ergonomic constraints. From this basic problem, we derive two major problem settings. The first is the problem of satisfying the physicians' ideal schedule as far as possible (or minimizing the total number of non-ideal scheduled duties) while not compromising on having the minimum number of unscheduled duties. The second is the setting where physicians do not provide their ideal schedule, but instead ergonomic constraints are employed across all physicians in minimizing the total number of unscheduled duties.

4 Mathematical Programming Models

The following notations are used in this paper.

Basic notations:

I = Set of physicians, $i \in \{1,2,\ldots,|I|\}$

J = Set of days, $j \in \{1,2,\ldots,|J|\}$

K = Set of shifts per day, $k \in \{1,2,\ldots,|K|\}$

L = Set of duties, $l \in \{1,2,\ldots,|L|\}$

The following sets are defined to facilitate definition of constraints:

L^C = {$l \in L$: l = duty with the limited number of resources available}

$L^{C'}$ = {$l \in L$: l = duty without the limited number of resources available}, while $L^C \cup L^{C'} = L$

L^H = {$l \in L$: l = *heavy* duty}

PRA = {$(i, j, k) \in I \times J \times K$: (i, j, k) = physician i requests not being assigned on day j shift k}

The following data parameters are used:

R_l = number of resources required to perform duty l ($l \in L^C$)

C_{jkl} = number of resources available for duty l on day j shift k ($j \in J$, $k \in K$, $l \in L^C$) (i.e. resource capacity)

A_{il} = number of duty l requested by physician i in a weekly schedule ($i \in I$, $l \in L$)

F_{ijkl} = 1 if physician i requests duty l on day j shift k, 0 otherwise

The decision and auxiliary variables are used:

X_{ijkl} = 1 if physician i is assigned to duty l on day j shift k, 0 otherwise

d_{ijkl}, d'_{ijkl} = deviations between the actual and ideal schedules of physician i to duty l on day j shift k ($d_{ijkl} \geq 0, d'_{ijkl} \geq 0$), related as follows:

- if duty l of physician i can be scheduled on day j shift k as requested in the ideal schedule, then $d_{ijkl} = 0$ and $d'_{ijkl} = 0$ (i.e. no deviation);
- if duty l of physician i cannot be scheduled on day j shift k as requested in the ideal schedule, then $d_{ijkl} = 1$ and $d'_{ijkl} = 0$;
- if duty l of physician i is scheduled on another day or shift, then $d_{ijkl} = 0$ and $d'_{ijkl} = 1$.

D_i = sum of deviations of physician i (see equation (17))
U_i = number of unscheduled duties of physician i
N_i = number of non-ideal scheduled duties of physician i (for Model IIa only)

Our problem can then be formulated by the following mathematical models. Model I is the base model which computes the minimum number of unscheduled duties for the unconstrained problem. This value provides the upper bound on the total unscheduled duties for subsequent constrained models involving physician preferences (Model IIa) and ergonomic constraints (Model IIb).

[**Model I**]

Minimize $Z = \sum_{i \in I} U_i$ (1)

subject to:

$$R_l \times \sum_{i \in I} X_{ijkl} \le C_{jkl} \quad j \in J, k \in K, l \in L^C \quad (2)$$

$$\sum_{j \in J} \sum_{k \in K} X_{ijkl} \le A_{il} \quad i \in I, l \in L \quad (3)$$

$$\sum_{l \in L} X_{ijkl} \le 1 \quad i \in I, j \in J, k \in K \quad (4)$$

$$\sum_{l \in L} X_{ijkl} = 0 \quad (i, j, k) \in PRA \quad (5)$$

$$U_i = \sum_{l \in L} A_{il} - \sum_{j \in J} \sum_{k \in K} \sum_{l \in L} X_{ijkl} \quad i \in I \quad (6)$$

$$X_{ijkl} \in \{0,1\} \quad i \in I, j \in J, k \in K, l \in L \quad (7)$$

$$U_i \in Z^+ \quad i \in I \quad (8)$$

(1) is the total number of unscheduled duties that needs to be minimized. Constraint (2) is the resource capacity constraint (total number of resources required does not exceed total number of available resources per shift). (3) represents the number of duties allocated to each physician cannot exceed his contractual commitments. (4) ensures that each physician cannot be assigned more than one duty in any shift, while (5) ensures that no duty would be assigned to a physician during any shifts-off or days-off requested. (6) defines the number of unscheduled duties (which is to be minimized). (7) imposes the 0-1 restrictions for the decision variables X_{ijkl}, while (8) is the nonnegative integrality constraint for the decision variables U_i.

Model IIa extends the base model considering physicians' ideal schedule. Let U_i^* denote the optimal solution containing the number of unscheduled duties for physician i obtained by Model I. We constrain the number of unscheduled for each physician to not exceed this upper bound (see (10)). Model IIa seeks to then minimize the total number of non-ideal scheduled duties. The deviations between the ideal and the actual schedules for each physician are represented by decision variables d_{ijkl} and d'_{ijkl}, as shown in (16). The total number of non-ideal duties for each physician is calculated by equations (15)–(18). The rest of the constraints are identical to those of Model I.

[**Model IIa**]

Minimize $Z = \sum_{i \in I} N_i$ (9)

subject to:

$$U_i \le U_i^* \quad i \in I \quad (10)$$

$$R_l \times \sum_{i \in I} X_{ijkl} \le C_{jkl} \quad j \in J, k \in K, l \in L^C \quad (11)$$

$$\sum_{j \in J} \sum_{k \in K} X_{ijkl} \le A_{il} \quad i \in I, l \in L \quad (12)$$

$$\sum_{l \in L} X_{ijkl} \le 1 \quad i \in I, j \in J, k \in K \quad (13)$$

$$\sum_{l \in L} X_{ijkl} = 0 \quad (i, j, k) \in PRA \quad (14)$$

$$U_i = \sum_{l \in L} A_{il} - \sum_{j \in J} \sum_{k \in K} \sum_{l \in L} X_{ijkl} \qquad i \in I \qquad (15)$$

$$X_{ijkl} + d_{ijkl} - d'_{ijkl} = F_{ijkl} \qquad i \in I, j \in J, k \in K, l \in L \qquad (16)$$

$$D_i = \sum_{j \in J} \sum_{k \in K} \sum_{l \in L} \left(d_{ijkl} + d'_{ijkl} \right) \qquad i \in I \qquad (17)$$

$$N_i = (D_i - U_i)/2 \qquad i \in I \qquad (18)$$

$$X_{ijkl}, d_{ijkl}, d'_{ijkl} \in \{0,1\} \qquad i \in I, j \in J, k \in K, l \in L \qquad (19)$$

$$U_i, N_i, D_i \in Z^+ \qquad i \in I \qquad (20)$$

The second problem setting we consider is where physicians do not express their ideal schedule but instead ergonomic constraints are imposed which aim to improve the quality of the schedule of each physician, formulated as Model IIb. Here, duties are classified into two different groups: *heavy* and *light* duties. A physician assigned to a *heavy* duty in a particular shift cannot be assigned to different *heavy* duties in the next shift on the same day, as represented by (27). If a physician is assigned to a *heavy* duty in the last shift on a particular day, he cannot also be assigned to different *heavy* duties in the first shift on the next day (see (28)). This condition implies that physicians might perform the same heavy duties in consecutive shifts.

[**Model IIb**]

Minimize $Z = \sum_{i \in I} U_i$ (21)

subject to:

$$R_l \times \sum_{i \in I} X_{ijkl} \le C_{jkl} \qquad j \in J, k \in K, l \in L^C \qquad (22)$$

$$\sum_{j \in J} \sum_{k \in K} X_{ijkl} \le A_{il} \qquad i \in I, l \in L \qquad (23)$$

$$\sum_{l \in L} X_{ijkl} \le 1 \qquad i \in I, j \in J, k \in K \qquad (24)$$

$$\sum_{l \in L} X_{ijkl} = 0 \qquad (i, j, k) \in PRA \qquad (25)$$

$$U_i = \sum_{l \in L} A_{il} - \sum_{j \in J} \sum_{k \in K} \sum_{l \in L} X_{ijkl} \qquad i \in I \qquad (26)$$

$$X_{ijkl_1} + X_{ij(k+1)l_2} \le 1 \qquad i \in I, j \in J, k \in \{1,2,\ldots,|K|-1\}, l_1 \,\&\, l_2 \in L^H (l_1 \ne l_2) \qquad (27)$$

$$X_{ij|K|l_1} + X_{i(j+1)1l_2} \le 1 \qquad i \in I, j \in \{1,2,\ldots,|J|-1\}, l_1 \,\&\, l_2 \in L^H (l_1 \ne l_2) \qquad (28)$$

$$X_{ijkl} \in \{0,1\} \qquad i \in I, j \in J, k \in K, l \in L \qquad (29)$$

$$U_i \in Z^+ \qquad i \in I \qquad (30)$$

5 Computational Results

In this section, we report a comprehensive suite of experimental results which aims to provide computational perspectives on one hand, and insights to hospital administrators.

5.1 Experimental Setup

First we discuss how problem instances were generated. The 6 sets of random instances were generated with varying values of the following parameters - total percentage of *heavy* duties assigned to physicians (last column of Table 1) and number of resources available in every shift (Table 2). The rationale behind this scenario is to conduct the sensitivity analysis that would be described more detail after the results reported. Other parameters, such as number of *heavy* duties and number of duties with limited resource capacity are set to constant. Besides random instances, we also provide a real case study provided by the Surgery

Department of a large local government hospital. Table 1 summarizes the characteristics of each problem set.

Table 1. Characteristics of Problem Instances

Problem Set	Number of physicians	Number of shifts per day	Number of days	Number of duties	Number of *heavy* duties	Number of duties with limited capacity	Total percentage of *heavy* duties*
Case study	15	2	5	9	3	3	73%
Random 1	20	2	5	7	3	3	20%
Random 2	20	2	5	7	3	3	30%
Random 3	20	2	5	7	3	3	40%
Random 4	20	2	5	7	3	3	50%
Random 5	20	2	5	7	3	3	60%
Random 6	20	2	5	7	3	3	70%

$* = \left(\sum_{i\in I}\sum_{l\in L^H} A_{il} / |I|\times|J|\times|K|\right)\times 100\%$

In order to generate sufficient hard problem instances for the purpose of sensitivity analysis, the resource capacity has to be carefully set, which we explain as follows. The upper bound for number of available resources for each random problem set is set as follows:

$$C_{jkl} = \left\lceil \frac{const \times R_l \times \sum_{i\in I} A_{il}}{|K|\times|J|} \right\rceil \qquad j \in J, k \in K, l \in L^C \qquad (31)$$

where *const* is a scale factor.

For each problem set, we generate different instances by varying the values of C_{jkl} as follows. For each duty l with the highest total number of resources required, the value of *const* is set to 2, and the value of C_{jkl} decreases by one unit until it is equal to $\left\lceil \frac{R_l \times \sum_{i\in I} A_{il}}{|K|\times|J|} \right\rceil - 1$ (for $j \in J, k \in K$). For other duties, the value of C_{jkl} is set from $\left\lceil \frac{R_l \times \sum_{i\in I} A_{il}}{|K|\times|J|} \right\rceil + 1$ to $\left\lceil \frac{R_l \times \sum_{i\in I} A_{il}}{|K|\times|J|} \right\rceil - 1$.

Table 2. Examples of varying values of C_{jkl} (Random 1 and Random 2 instances)

Problem Set	Instances	L^c		
		Duty 1	Duty 2	Duty 3
Random 1		15	28	22
	Random 1a	3	6	4
	Random 1b	3	5	4
	Random 1c	3	4	4
	Random 1d	3	3	4
	Random 1e	3	3	3
	Random 1f	2	3	3
	Random 1g	1	2	2
Random 2		21	46	32
	Random 2a	4	10	5
	Random 2b	4	9	5
	Random 2c	4	8	5
	Random 2d	4	7	5
	Random 2e	4	6	5
	Random 2f	4	5	5
	Random 2g	4	5	4
	Random 2h	3	5	4
	Random 2i	2	4	3

As an illustration, Table 2 presents three columns that give the varying values of C_{jkl} for different duties defined in the instances of the Random 1 and Random 2 problem sets. For Random 1, the total number of resources required for Duty 1, Duty 2 and Duty 3 are 15, 28 and

22, respectively. Here, Duty 2 is the duty with the highest number of resources required and hence the value of C_{jkl} for Duty 2 is set to $\left\lceil \frac{2\times 28}{2\times 5} \right\rceil$ and decreases by one unit until it is equal to $\left\lceil \frac{28}{2\times 5} \right\rceil - 1$. For Duty 1 and Duty 3, we set the initial values of C_{jkl} to $\left\lceil \frac{15}{2\times 5} \right\rceil + 1$ and $\left\lceil \frac{22}{2\times 5} \right\rceil + 1$, respectively and varying the values according to the description given above. For instance, for Duty 1, by varying the values for C_{jkl}, there are 7 different problem instances generated.

In the following, we report a suite of computational results and analysis obtained from our mathematical models described above. Our mathematical models were implemented using ILOG OPL Studio 5.5 and executed on a Intel (R) Core (TM)[2] Duo CPU 2.33GHz with 1.96GB RAM that runs Microsoft Windows XP.

5.2 Results from Model I and IIa

First, results obtained from **Model I** for Random 1 and Random 2 problems are shown in Table 3. It is interesting to observe a two-point phase transition in the minimum number of unscheduled duties (column 2) with changing values of C_{jkl}. It remains unchanged over a sufficiently large range of values. As the value of C_{jkl} of the duty with the highest requirement tends to $\left\lceil \frac{R_l \times \sum_{i\in I} A_{il}}{|K|\times|J|} \right\rceil$, the number of unscheduled duties starts to increase. For example, as we decrease the C_{jkl}'s value for Duty 2 from 6 to 4 for Random 1 instances, the number of unscheduled duty remains zero, but when this value reaches 3, the number of unscheduled duties increases to 4. Then, when the number of resources available is set to $\left\lceil \frac{R_l \times \sum_{i\in I} A_{il}}{|K|\times|J|} \right\rceil - 1$ for each activity $l \in L^C$, the number of unscheduled duties increases drastically from 5 to 10 unscheduled duties. The same behavior was also observed for the other problem sets listed in Table 1.

From Table 3 again, we observe that the number of unscheduled duties is very low with respect to the number of scheduled duties. The percentage of unscheduled duties is on average less than 3% for Random 1 and Random 2 instances. Similar observations are made for other random problem sets. These results demonstrate the effectiveness of the optimization model on random hard instances. It is interesting to see these results in the light of the real-world case study problem instance where the percentage of unscheduled duties obtained is around 4.7%. This gives evidence to the hospital management that their resource capacity has reached a critical threshold as typified by problem instances 1g and 2i, and consequently towards a drastic reduction of performance if resources cannot come up to par with physician duties.

Table 3 also presents results on the extent of satisfiability of ideal schedule by the actual schedule, obtained from running **Model IIa**. These results are summarized in Figure 2, which visualizes the effect. Note that the left and right y-axes are on different scales, and measures the percentage of ideal, non-ideal scheduled and unscheduled; and the resource availability/requirement gap[i] %*Gap* respectively. From the figure, we can infer that in order to ensure zero unscheduled duties (which is often a hard constraint, since doctor duties should not be unfulfilled), the total number of available resources for each activity $l \in L^C$ must be above the threshold $\left\lceil \frac{R_l \times \sum_{i\in I} A_{il}}{|K|\times|J|} \right\rceil + 1$ (see instances 1a, 1b and 1c). We can also infer that when the

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[i] The resource availability/requirement gap is defined as $\%Gap = \frac{|K|\times|L|\times\sum_{j\in J}\sum_{k\in K}\sum_{l\in L^C} C_{jkl} - \sum_{i\in I}\sum_{l\in L^C} R_l \times A_{il}}{\sum_{i\in I}\sum_{l\in L^C} R_l \times A_{il}} \times 100\%$. It represents resource buffer, i.e. the proportion of total resource availability that exceeds the sum of resource requirement requested.

%*Gap* is decreased below 23%, the percentage of unscheduled duties will be doubled (see instances 1f vs 1g, also 2h vs 2i). Similar observations have been made for the rest of the problem instances. From the hospital administration standpoint, the latter result shows the critical resource availability threshold below which the degradation of service performance will be keenly felt.

Table 3. Summary of results for Models I and IIa

Problem Instances	Number of unscheduled duties	Number of scheduled duties		Percentage of unscheduled duties (%)	Percentage of scheduled duties (%)	
		Ideal	Non-ideal		Ideal	Non-ideal
Case study	7	139	4	4.7	92.6	2.7
Random 1a	0	196	4	0	98	2
Random 1b	0	192	8	0	96	4
Random 1c	0	192	8	0	96	4
Random 1d	4	186	10	2	93	5
Random 1e	5	179	16	2.5	89.5	8
Random 1f	5	178	17	2.5	89	8.5
Random 1g	10	173	17	5	86.5	8.5
Random 2a	0	196	4	0	98	2
Random 2b	0	196	4	0	98	2
Random 2c	0	196	4	0	98	2
Random 2d	0	194	6	0	97	3
Random 2e	0	194	6	0	97	3
Random 2f	3	184	13	1.5	92	6.5
Random 2g	3	184	13	1.5	92	6.5
Random 2h	3	183	14	1.5	91.5	7
Random 2i	10	176	14	5	88	7

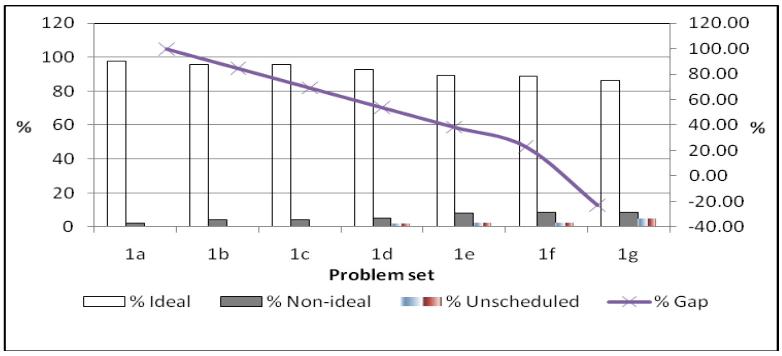

Figure 2. Parameter analysis of Random 1 Problem Set

Next, the result of the case study problem instance is compared with that of the actual allocation generated manually by the hospital, as summarized in Table 4. Although the number of unscheduled duties via manual allocation is better than the results obtained by Model IIa, the number of non-ideal scheduled duties is significantly higher than that of the model solution. We also found that two physicians have to cancel their days-off or shifts-off for other duties. This outcome is very undesirable since they might have external commitments that cannot be delayed or cancelled.

Table 4. Comparison between the manual allocation and model solution on a real case

	Manual allocation	Optimal solution
Number of unscheduled duties	5	7
Number of non-ideal scheduled duties	10	4
Number of physicians assigned duties during days-off or shifts-off	2	0

5.3 Results from Model IIb

Table 5 presents results obtained by running Model IIb against our problem instances. We observe that only Random 6 instances (where the percentage of *heavy* duties reaches 70%)

could not be solved to optimality within the computation time limit imposed. As such, we only report the best known solutions that could be obtained for these problem instances.

Table 5. Computational results of Model IIb

Problem instance	Number of unscheduled duties	Number of scheduled duties
Case study	8	142
Random 1a	0	200
Random 1b	0	200
Random 1c	0	200
Random 1d	4	196
Random 1e	5	195
Random 1f	5	195
Random 1g	10	190
Random 2a	0	200
Random 2b	0	200
Random 2c	0	200
Random 2d	0	200
Random 2e	0	200
Random 2f	3	197
Random 2g	3	197
Random 2h	3	197
Random 2i	10	190
Random 3a	0	200
Random 3b	0	200
Random 3c	0	200
Random 3d	0	200
Random 3e	0	200
Random 3f	5	195
Random 3g	9	191
Random 3h	9	191
Random 3i	10	190
Random 4a	0	200
Random 4b	0	200
Random 4c	0	200
Random 4d	0	200
Random 4e	0	200
Random 4f	0	200
Random 4g	0	200
Random 4h	3	197
Random 4i	6	194
Random 4j	6	194
Random 4k	10	190
Random 5a	0	200
Random 5b	0	200
Random 5c	0	200
Random 5d	0	200
Random 5e	0	200
Random 5f	0	200
Random 5g	0	200
Random 5h	0	200
Random 5i	0	200
Random 5j	3	197
Random 5k	8	192
Random 5l	8	192
Random 5m	10	190
Random 6a*	2	198
Random 6b*	2	198
Random 6c*	3	197
Random 6d*	3	197
Random 6e*	3	197
Random 6f*	3	197
Random 6g*	3	197
Random 6h*	3	197
Random 6i*	4	196
Random 6j*	4	196
Random 6k*	5	195
Random 6l*	6	194
Random 6m*	7	193

Problem instance	Number of unscheduled duties	Number of scheduled duties
Random 6n*	7	193
Random 6o*	23	177

*CPU time = 6 hours

Finally, using Model IIb, we perform sensitivity analysis of ergonomic constraints on schedule quality, which is of interest to the hospital administrator. These experiments are conducted over the same set of random instances as shown in Table 1 (see last column), and the findings are presented in Figure 3. The x-axis represents the percentage of heavy duties.

(a) The **square-dotted curve** shows the minimum total resource that should be available in order that the number of unscheduled duties will remain zero, while the **diamond-dotted curve** shows the corresponding resource requirement given by the input (refer to left y-axis). Comparing these curves we observe that across the board, a resource buffer of 20% of the total resource requirement should be set aside, in order to satisfy all ergonomic constraints. In other words, the cost of enforcing ergonomics constraints is 20% of resource requirement.

(b) Another observation is the impact on the number of unscheduled duties under fixed resource constraints. The **triangular-dotted curve** shows the number of unscheduled duties for problem instance 1c (refer to right y-axis). Note that as we increase the percentage of *heavy* duties, the total number of unscheduled duties increases gradually, until a certain threshold of 40%, when it is observed to increase sharply. This phase transition phenomenon is also observed in other random instances. It provides insights to the hospital administrator in terms of planning the limits of *heavy* duties for the physicians.

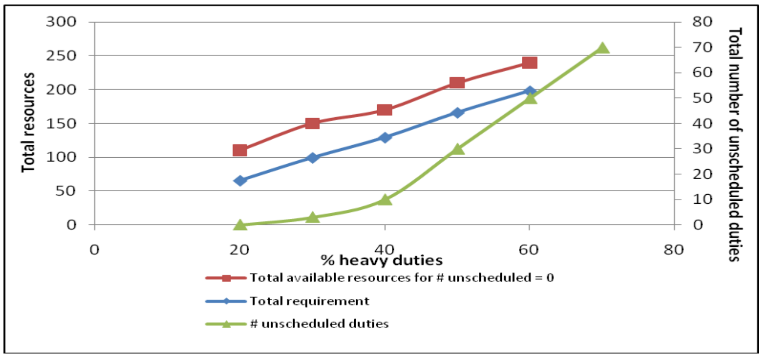

Figure 3. Sensitivity analysis on percentage of *heavy* duties

6 Conclusion

In this paper, we study the master physician scheduling problem experimentally. To the best of our knowledge, this is the first attempt that looks holistically at an entire range of physician duties quantitatively that enables hospital administrations to cope with increasingly stringent patient demands. Since the particular problem studied is representative of the Surgery Department of a large local government hospital, we believe our model requires minor customizations for use in other hospitals with similar constraints and preference structures.

We see many possibilities of extending the work. Our approach in this paper is purely optimization-based, including the handling of physician preferences. It will be interesting to investigate how other preference-handling methods (such as CP-nets) can be incorporated to model complex physician preferences. Similarly, one might also consider fairness constraints commonly seen in hospitals (Gendreau et al., 2007). Algorithmically, it would be interesting to tackle large-scale problems (such as the Random 6 instances) that cannot be solved by exact optimization models with meta-heuristic or evolutionary approaches, or investigate multi-objective approaches since the problem can be modelled as a multi-objective problem.

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